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Abstract

The design of urinals for public and semi-public restrooms is difficult to standardise, as they are not intended for single users and must accommodate a wide range of anatomical and behavioural variability. Beyond technical specifications, effective design must account for social norms related to privacy as well as hygiene and sanitation requirements. In particular, the height, shape, and dimensions of a urinal basin should minimize splashing, spillage, and dribbling beyond the rim. An adequately designed basin edge is therefore essential, and univariate or bivariate Gaussian models have traditionally been used to shape urinal design. The primary objective of this study is to examine how Gaussian distributions and heavier-tailed tanBeta distributions, such as the Cauchy distribution, which is the tangent of a Uniform random variable, can contribute to improved urinal design aimed at improving floor tidiness. We extend this approach by analysing the tangent transformations of the symmetric, Beta(1/2,1/2) and Beta(2,2) distributions. By comparing their central regions containing 68% probability, we assess their implications for basin geometry. Aside from statistical modelling, critical appraisal of statistical methodology on sampling by interviewing is considered in this multidisciplinary study, that also deals with the relevance of anatomical characteristics for the design of these indispensable urbanism commodities. Results suggest that successful urinal design requires a balance between aesthetic appeal and ergonomic effectiveness, while accounting for the substantial diversity in male anatomical and biometric characteristics, since the prevalence of some defects indicate that wider models would be more fit to ensure tidiness.

Keywords: Human male biometry; Normal distribution; Cauchy distribution; Beta and tanBeta distributions; Urinals design; Ergonomics; Metrology

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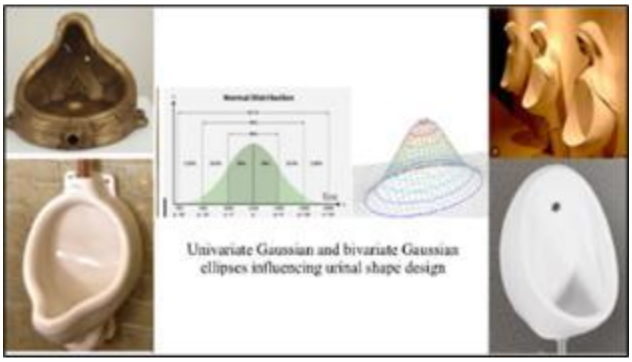
Graphical Abstract



1. Introduction

Men's toilets and restrooms constitute essential yet often overlooked elements of urban infrastructure. While water closets are subject to detailed regulatory oversight — exemplified by the European Standard EN 997 [1], approved by the Comité Européen de Normalisation in 2018 — no equivalent European Nor exists for urinals. This absence may be attributed to the considerable variability of social norms, moral expectations, and usage patterns across countries.

In some national contexts, guidance has nevertheless been developed. For example, in the United Kingdom the Ministry of Works and Human Settlement issued recommendations in 2021 based on the principle of "*leaving no one behind*" [2]. Such initiatives highlight the growing recognition of inclusivity and accessibility in sanitary design, while also underscoring the lack of harmonised standards at the European level.

Despite the diversity of architectural solutions and cultural practices, a fundamental social and sanitary requirement remains universal: restroom floors must be kept clean and dry. Achieving this objective depends not only on technical design choices but also on user behaviour in shared facilities. Proper behaviour should be strictly observed by all users. In Figure 1 (left), an ephebe in ancient Greece prevents the intoxicated poet from urinating on the floor while declaiming.

Figure 1 Please comply with this request instead of engaging in foolish competitions, either poetic or athletic!

Individuals with competitive tendencies should feel discouraged to engage in exhibitionism contests, that almost certainly will dirty the floor. In practice, deviations from appropriate conduct — whether due to inattentiveness, misuse, or exhibitionist tendencies — can compromise hygiene. Consequently, urinal design must seek to mitigate these risks through ergonomic, geometric, and functional features, complementing metrological normative guidance and promoting acceptable use.

Trough urinals are now largely prohibited, and contemporary facilities typically employ individual wall-hung or floor-mounted urinals. The design of these fixtures — including geometry, installation height, and overall dimensions — should be optimised to minimise splashing and spillage. Nevertheless, substantial variability persists across manufacturers with respect to dimensions and installation guidelines.

Common urinal widths range from 300 mm to 450 mm. Recent stylistic trends favour slimmer profiles than those of traditional models, such as the *Hidra Drop* (270 mm wide) or the *Starck 3* urinal designed by Philippe Starck (245 mm wide), shown in Figure 2.

Figure 2 *Hidra Drop* (left), anti-spilling *Cornucopia* and *Tear Drop* urinals (middle) and *Starck 3* (right) urinals

Manufacturers of these products claim anti-splash performance, with their reduced rim depth tend to encourage urination from very close range. This practice can raise concerns regarding hygiene and disease transmission. It should be also noted that certain modern urinal shapes and rim design may be prone to splashing and therefore require particularly careful use.

Splashing remains a significant issue for many user groups. For example, in Islamic practice, contact with urine droplets constitutes impurity (*najasah*), which invalidates ritual prayer. Such considerations further emphasise the importance of urinal designs that reliably minimise splash and floor soiling across diverse cultural and sanitary contexts.

Although several stylish and unconventional urinals exist, they remain notable exceptions to the two traditional geometries illustrated in Figure 3.

Figure 3 From left to right, Neo-baroque Vineta Urinal, Marcel Duchamp's Fountain, nuns urinals in Sister Sledge restaurant in the Soho, London, Clark Sorensen's Hollyhock and Morning Glory Urinals

From a statistical perspective, these dominant forms closely resemble either univariate or bivariate Gaussian (normal) distributions. The frontal rims of the *Vineta Urinoir* and Duchamp's *Fountain* appear to be inspired by the central region of a univariate Gaussian probability density function. In contrast, the rim of the receptacle in the so-called "*nuns' urinal*" evokes a probability concentration contour of a bivariate Gaussian distribution. Similarly, the opening rims of many luxurious sculptural urinals, presented in Section 6, approximate bi-Gaussian probability contours. This resemblance is particularly evident in the Hollyhock and Morning Glory urinals shown in Figure 3 (right).

Designers understand that adhering to these shapes helps minimise urine splashing onto the floor. Ergonomic considerations, together with tradition, explain why most manufacturers continue to base urinal models on Gaussian-inspired geometries.

Section 2 provides general background, from the emergence of public toilets around 1850 to social etiquette considerations in restroom planning and design. Consensual directives on minimum depth, separation spaces, clearance around each urinal and urinal rim height, indicated therein, derive from statistical knowledge on users' bodily characteristics and social behaviour.

A quotation of a droll character from Graham Greenes [3] *Travels with my Aunt* description of his self-appointed investigation on his urination frequency and duration, though risible, describes nonetheless a layman's very accurate understanding of the uses of Statistics in knowledge building.

Effective design and standardisation should consider biometric data, including population age distribution, average height, leg length, and body proportions. Considering the importance of ergonomics, Section 3 examines how real male anatomy often departs from idealised forms in classical art, particularly regarding scrotal and penile features; it is also discussed whether self-reported dimensions overestimate penile length, or instead clinician measurements underestimate it due to embarrassment — or both. Appendix A, reproduced from [4], describes the stages of male sexual development. Statistics on the prevalence of non-standard sexual anatomy, including enlarged prostate, penile curvature, hypospadias, Peyronie's disease, and phimosis contribute to the urine stream striking the urinal at a steep angle, increasing the propensity to splash urine, are also indicated in Section 3. Those statistics are mainly unreliable — ranging from 0.3% to 20% in what regards Peyronie's — due to many factors, namely the use of convenience samples instead of random samples, deriving from patient's embarrassment. Nevertheless, they contribute to the overall belief that a norm requiring larger rims would benefit reduction of urine spilling and splashing, improving hygiene and public health.

Section 4 discusses the role of urinal shape and dimensions in hygiene, referencing statistical models. While designers often seem to implicitly assume that the urine stream follows a Gaussian distribution, a more realistic approach is to consider the hitting point as uniformly distributed. The tangent of a [-π/2,π/2] uniform random variable is standard Cauchy distributed; hence, adopting a bivariate Cauchy distribution to define concentration ellipses and rim dimensions offers a practical alternative. The Uniform is the simplest Beta random variable. We also investigate tangent transformations of other tractable [-π/2,π/2] symmetric Beta distributions, the ArcSin Beta(1/2,1/2) and the Logistic Parabola Beta(2,2), to assess their possible implications for urinal design.

Section 5 explores additional modelling considerations, incorporating statistical data on male anatomy from Section 3. New AI tools such as *Pixlr Express* can use the quantile function of the tanBeta models developed in Section 4 can be used at the design stage for a compromise of elegance/beauty with functionality for tidiness. Final remarks on users' behaviour, and on how it can be guided or accommodated choosing unexpected innovative designs, such as repurposed kegs in pubs. Urinal design remains an active and diverse field, with some luxury sculptural models valued around USD 20,000. Notable examples are provided in Section 6.

Despite the importance of the subject in Urbanism and Social Welfare, deserving the multidisciplinary focus we adopted, it is to a certain extent weird and potentially boring, tempting us to occasionally use some humour and irreverence when commenting illustrations and human behaviour.

2. Some General Background

Pissoirs were introduced in several major European cities during the nineteenth century to reduce public urination on building façades and streets. In 1843, Comte Rambuteau, the préfet of the Department of the Seine, ordered the installation of four hundred colonnes Rambuteau (Figure 4, left), individual street urinals distributed throughout Paris. Disliking the use of his own title for the devices, Rambuteau proposed the term *vespasienne*, invoking the Roman Emperor Vespasian, who had famously levied a tax on urine, which in the Roman Empire was then used for tanning and laundering.

Figure 4 Colonne Rambuteau, a Paris vespasiennes, and Alfred Hitchcock wandering around a vespasienne while being interviewed by the French television in 1960

Roger Peyrefitte [5], in *Des Français*, went so far as to claim that vespasiennes were Parisian icons, as emblematic as the Eiffel Tower. He recalled that Alfred Hitchcock, when interviewed for the French television in 1960, was filmed wandering nonsensically around a rusty vespasienne (Figure 4, right). By the way, "to hitch" means "to move by jerks or with a tug", and the noun "hitch" means "limp", "jerk", "stoppage".

In popular culture, aside from their hygienic purpose, urinals were discreet meeting places, as depicted in the BBC comedy series *'Allo 'Allo!*, where the Nouvion vespasienne was a meeting place for René Artois and members of the Resistance. In one of the episodes, that vespasienne was blown up when Officer Crabtree was in it, which seems appropriate, since with is exquisite accent he frequently claimed to be "*an agent of the Loo*" (agent of the Law) and announced that he was "*pissing by*" (passing by). Other discreet or indiscreet meetings also occur often in public urinals.

The humourist Alphonse Allais [6] recounted his longing for Parisian vespasiennes during a visit to London. Having consumed large quantities of ale, stout and porter, he lamented that Tottenham Court Road lacked this essential public amenity, nostalgically recalling the vespasienne on Boulevard Montmartre. To resolve his urgent situation, Allais entered a chemist's shop and requested a urine analysis. Finding the provided container too small, he relieved himself in a pot that was simmering on the stove, before departing discreetly (*filer à l'anglaise*) to return to Paris, "a capital graced with innumerable vespasiennes".

Urban planning has evolved, and today public toilets are required in airports, transportation hubs, and essential facilities such as hospitals. Semi-public restrooms in shopping malls, restaurants, and pubs must likewise be designed with user convenience in mind. There remains substantial variability in urinal dimensions and installation guidelines. A widely accepted requirement specifies that urinals must have a minimum depth of 345 mm, measured from the outer rim to the back of the fixture, allowing users to stand close enough while minimising spills and discomfort. There should be a clearance of about 150 mm on each side of the urinal and 600 mm in front of the unit to ensure comfort (ideally an unobstructed area of about 120cm×76.6cm in front of it), and the rim must be no higher than 430 mm above the finished floor, so wheelchair users can reach it without difficulty, since a higher installation creates an unnecessary barrier. There should be a minimum of 38 cm centre-to-centre between adjacent urinals.

There are consensual etiquette and social norms in public toilets, intended to preserve privacy, such as maintaining physical distance, keeping quiet unless conversing with a companion, avoiding eye contact, and refraining from exhibitionism as well as from peeping. For this purpose, some urinals are installed with separations. However, guidelines on separations remain flexible: urinals may be installed with or without partitions, typically spaced between 38-40 cm apart. Even when partitions are present (see Figure 5), some designs allow a degree of visual intrusion due to transparent materials or insufficient height, which makes possible looking over them.

Figure 5 Urinals partitions

Eventually, in non-public urinals, the absence of partitions may facilitate more intimate contact, for instance in special pub gents. Some small and narrow partitions seem appropriate to satisfy both users who prefer greater privacy and those who are less concerned with consequences of visual exposure. Moreover, they may persuade some shy users to urinate from close range, minimising urine spilling to the floor (Figure 6). On the other hand, modern street portable urinal designs use very small rim, forcing users to lean very closely to the pissoir, for public decency purposes (Figure 6, right).

Figure 6 Urinals without partition or with small inefficient partition. Portable street urinal

Knowledge about the urinal behaviour is scarce. An important aspect of planning and designing toilets and restrooms is estimating the average and maximum number of users expected to require the facilities. Decisions regarding the number of urinal stands should be informed by statistical data, including the frequency of urination — which can vary with age, health conditions such as diabetes, or the use of diuretics — and the typical duration of urination. Interestingly, Graham Greene [3], in *Travels with My Aunt*, illustrates a layperson's surprisingly accurate approach to collecting urination statistics. The funny character O'Toole explains to the narrator: "*I count while I'm pissing and then I write down how long I 've taken and what time it is*". From his July 28 data

7.15	0.17
10.45	0.37
12.30	0.50
13.15	0.32
13.40	0.50
14.05	0.20
15.45	0.37
18.40	0.28
10.30	? Forgot to time
	4 m. 31 sec.

he draws the conclusion that "*we spend more than one whole day a year pissing*".

Although it is a a convenience, non-random sample, it is nonetheless possible to infer some useful information about users' behaviour and facility requirements. His rationale is clever and statistically sound, but from the fact that the data he uses is not a random sample from the population. He also adds that "*You've only got to multiply by seven. That makes half an hour a week. Twenty-six hours a year.*" — thus mimicking the estimation of a total (cf. Scheaffer et al. [7], Chapter 4, namely p. 87).

However, O'Toole doesn't completely miss the point that he is not using a proper average as a starting point ("*Of course shipboard life isn't quite average*"), and that his collection of data may contain outliers ("*Look at this time here — one minute, fifty-five seconds. That's more than the average, but then I 've noted down two gins.*"), eventually due to correlated covariables, and that there are missing data (*"Forgot to time"; and "n.c. — that stands for 'not complete.' I went out to dinner in BA* [Buenos Aires] *and left my notebook at home*.").

Moreover, he acknowledges that "*There's a lot of variations too I haven't accounted for*", highlighting the role of confounding factors in statistical observations. His final observations on the comparison of urination patterns of sick patients and of healthy subjects seems common sense intuition of the fundamental principles of experimental design as formalized by Fisher's *Design of Experiments* [8].

3. The Human Male Body, and Penile and Scrotal Anatomy

The Greek sculptor Polykleitos (fifth century BCE), through his ideally proportioned bronze sculpture Doryphoros and his *Canon* [9], exerted a lasting influence on the belief that the ideal proportions of the male body should conform to the "eight-heads" rule, as illustrated in Figure 7 (eight-heads-high male figure, after Hamm [10], p. 39).

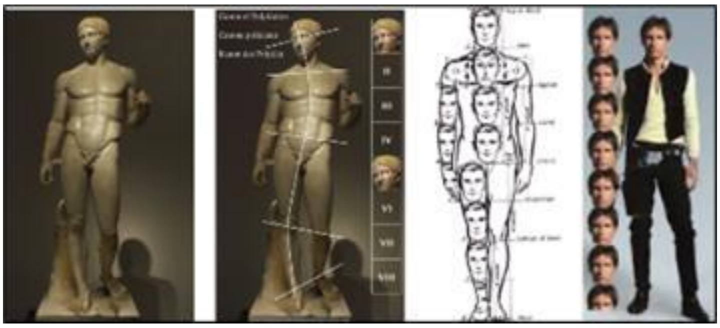


Figure 7 Grecian Polykleitos Canon

The Polykleitos Canon:

- *1st Head count:* starts from the top of the head to the bottom of the chin.
- *2nd Head count:* ends under the armpit, usually includes the shoulder and the neck in this.
- *3rd Head count:* ends at the navel area, under the ribcage.
- *4th Head count:* ends at the crotch/groin area.
- *5th Head count:* ends at the tip of the fingers if the hands are place straight down or near the hip.
- *6th Head count:* ends under the knees.
- *7th Head count:* ends slightly in the middle of the shin, under the calf muscles.

- *8th Head count:* calf muscles — ends at the feet.

Leonardo da Vinci's Vitruvian Man [11] and Michelangelo's David (see Figure 8) are celebrated as two icons of male bodily perfection (at least until David enjoyed fast food at an USA museum cafeteria, rightmost photo in Figure 8, created by the advertising agency Scholz & Friends, Germany, for a campaign promoting healthy behaviour), although none of them follows the strict Grecian Polykleitos *Canon*.

Figure 8 Mature humane male dimension archetypes.

Moreover, it is ideally considered that the proportions (soles of feet to navel)/(top of the head to navel and shoulders waist) should ideally approximate the ubiquitous Fibonacci (Leonardo de Pisa, circa 1170 to circa 1240-50) golden ratio $\phi= (1+\sqrt{5})/2 \approx 1.618$, which fascinates artists and architects.

In da Vinci's Vitruvian Man (circa 1492), the above ratio is approximately 1.656, slightly higher than ϕ. Michelangelo's David (1504) deliberately exaggerates certain proportions to enhance visual impact, emphasising strength, and compensate foreshortening when viewed from a distance and from below. Della Monica et al. [12] offer a detailed comparison between the Vitruvian Man and David (Figure 9), highlighting the subtle differences in these masterpieces of human form.

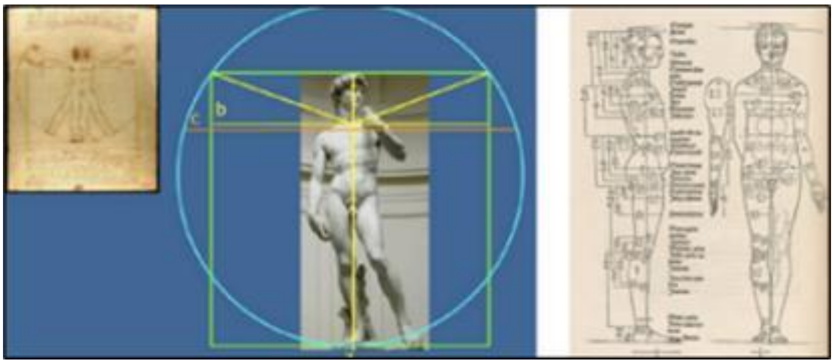


Figure 9 David and the Vitruvian Man proportions. (Source: Della Monica et al. [12].). Dürer engraving

Albrecht Dürer (1471–1528) provides a more detailed and realistic examination of human bodily variability in *De Varietate Figurarum* (*Four Books on Human Proportion*), published posthumously in Nuremberg in 1532 and 1534, see Figure 9 (right) and commentaries in Gallucci [13].

As observed by Morris [14], contrarily to what happens with the other primates, the penis of naked men is clearly visible due to the standing position. On the other hand, testicles should roll over one another when crossing the legs, to avoid discomfort or even pain, and therefore the right testicle is higher than the left one. Therefore, usually the adult penis is a little bit left positioned, as seen in the two left photos in Figure 10.

Figure 10 Usual left inclined penis vs. central position in the Vitruvian Man and in Michelangelo's David

This fact is widely known by tailors [15], whose sewing patterns provide a small extra width to the left fly of trousers. Curiously, artists in general miss the point, for instance the Vitruvian Man and of Michelangelo's David carving icons of male perfection depict an almost symmetrically vertical positioned penis (see the two right-hand zooms in Figure 10).

Chris McManus [16] published a remarkable note on the scrotal asymmetry in man. See also his exceptional book McManus [17] on the vital role of asymmetry, winner of an Aventis Prize for Science Books. McManus noted that Greek statues' left testicles are always larger than the right ones, contrarily to what happens on real men.

By the way, prejudice and mores change with time. Nowadays, most men would like to have a long and gross penis (and there are even surgeries aimed at enlarging the penis size). In the past, a small penis was more fashionable. In fact, large penis and erections were considered ridicule, as plainly documented in Aristophane's comedy *Lysistrata* [18].

Figure 11 Priapus and satyr exaggerated penises. Bestial lovemaking of deities: Pan and a she-goat; Zeus disguised as a swan and Leda

Large penis were the attribute of satyric representations of Priapus and satyrs, as depicted in Figure 11, fit for bestial sex, as in the carving of the demigod Pan with a she-goat, found in Herculaneum, or the emblematic metaphoric long neck and gross glans-head of the Zeus-Swan loving Leda (Figure 11), illustrating Greco-Roman civilisation accepted bestial lovemaking of gods and demigods. We are free to raise the hypothesis that Proust's [19] *Un Amour de Swan*n echoes the compulsive desire of the Swan-Zeus for Leda that, once fulfilled, vanishes in thin air — Zeus rutting for Leda didn't last, nor his bull-breeding for Europe, or eagle-heat for Ganymede. Similarly, the last sentence of *Un Amour de Swan*n plainly states the evanescence of body infatuation: "*Et avec cette muflerie intermittente qui reparaissait chez lui dès qu'il n'était plus malheureux et qui baissait du même coup le niveau de sa moralité, il s'écria en lui-même: 'Dire que j'ai gâché des années de ma vie, que j'ai voulu mourir, que j'ai eu mon plus grand amour, pour une femme qui ne me plaisait pas, qui n'était pas mon genre!*'" (And with the old, intermittent caddishness which reappeared in him when he was no longer unhappy and his moral standards dropped accordingly, he exclaimed to himself: 'To think that I've wasted years of my life, that I've longed to die, that I've experienced my greatest love, for a woman who didn't appeal to me, who wasn't even my type!'" (English translation by C. K. Scott Moncrieff and Terence Kilmartin, Swann in Love, New York: Vintage Books; 1984.)) Besides Proust [20] considered that "*L'amour platonique est peu de chose* […] *l'amour sensuel* […] *moins encore*." (Platonic love is little thing […] sensual love even less.")

Contrary to popular assumptions, there is no evidence that supports a significant correlation between race and penis size. Results from a standard ANOVA show that the range of penis sizes within any racial group is far greater than the average difference between groups. Veale et al. [21] meta-analysis computed the length of flaccid, pendulous penis 9.16 ±1.57 cm and of the erect penis 13.12 ±1.66 cm, and the circumference of the flaccid penis 9.31 ±0.90 cm and of the erect penis 11.66 ±1.10. Veale et al. (2015) nomograms, see Figure 12, are more informative.

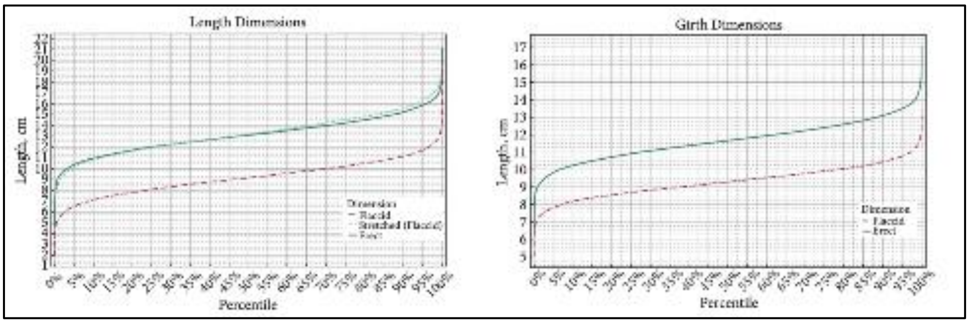


Figure 12 Length and girth dimensions nomograms. (Source: Veale et al. [21])

From them we can estimate extremes and quartiles, reported in Table 1, and display in Figure 13 the corresponding box-and whiskers plots.

Table 1 Extremes and Quartiles of lengths and girths of penises using Veale et al. [21] nomograms.

	min	Q_1	M	Q_3	max	Q_3-Q_1
Flaccid length	2.00	8.05	9-10	10.20	10.30	2.25
Erect length	4.70	12.05	13.05	14.25	21.20	2.20
Flaccid girth	5.00	8.75	9.30	9.95	13.50	1.20
Erect girth	6.40	10.90	11.65	12.45	17.10	1.55

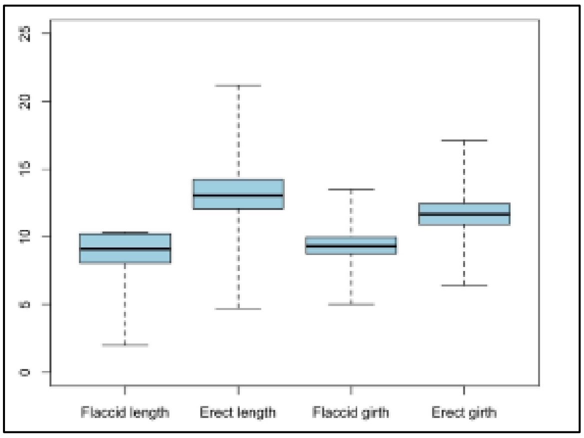


Figure 13 Length and girth of flaccid and of erect penises box-plots using Veale et al. [21] nomograms.

Belladelli et al. [22] performed a meta-analysis of 75 studies, encompassing 55,761 men, and reported the 95% confidence intervals for penis length, as shown in Table 2.

Table 2. 95\% confidence interval for the penis length in cm (source: Belladelli et al, [22]).

stretched length	[12.48,13.39]
erect length	[13.20,14.65]

Similarly, Mostafaei et al. [23] conducted a meta-analysis of 33 studies, including 36,883 men, and provided the corresponding 95% confidence intervals, as summarized in Table 3.

Table 3 95\% confidence interval for the penis length in cm (source: Mostafaei et al, [23]).

Flaccid length	[8.75,9.69]
stretched length	[12.21,13.47]
erect length	[12.00,15.68]

Measurements in the meta-analysed studies were obtained by clinicians. General knowledge assumes that studies relying on self-reported data often report larger averages than clinician-measured values, reflecting a tendency for overestimation.

Conversely, measurements taken by a professional are likely to be biased, analogous to the well-documented phenomenon in which blood pressure rises when measured by a clinician.

A well-known, though unpublished, study on the biometrical characteristics of the Bushmen of Angola, included measurements of penis length. At the time it was considered lewd visual contact between the female researcher and the penis of the San, so that concerns about decency required that two soldiers had a curtain between the female researcher and the subjects. Consequently, she had to grope to perform the measurement.

The participants were offered a chicken as an incentive for attending the measurements session. It was later discovered that some of them hid the reward and returned to be measured again and again to obtain more valuable food supplies.

At the end, the size of the sample exceeded the population size… Interestingly, measurements taken later in the study — presumably from individuals who had already participated, having been subject to manipulation — tended to be larger than those recorded in the initial phase

Furthermore, *a posteriori*, it was discovered that sample characteristics (average, median, top order statistics) of the measurements taken by the female professor were significantly higher than those obtained by her male assistant, suggesting potential differences in measurement technique or participant behaviour depending on the examiner, and attesting straight orientation of most of those San men.

This historical case study highlights potential sources of confounding, such as the effects of the examiner's characteristics and the subject's level of comfort or inhibition. Sexual orientation of the subject, and attractiveness of the clinician taking the measurements, may obviously have a bearing on the degree of sexual arousal, and therefore on the penis length and girth. Consequently, studies relying on clinician measurements may be prone to report biased estimation, just as studies based on self-reported measurements, and in fact it is controversial whether self-report tend to overestimate size or clinician's measurements result in underestimation.

To persuade users to avoid spilling urine on the floor, some designers display a blowfly in the urinal. Often it is placed in a central position, but some designers, aware that the usual penis position is a little bit left-oriented, place the blowfly target to the left, see Figure 14. In Sodoma bar (Reykjavik), instead of a fly, the photos of infamous bankers responsible for Iceland financial crash have been stuck on the urinals for customers good aim and satisfaction.

Figure 14 Blowfly aiming target in urinals. Toilets of the Sodoma bar in central Reykjavik

The inability to retract the prepuce covering the glans of the penis (phimosis) may impair the urine stream control. The systematic review of forty-three eligible studies of Morris et al. [24] reports that for adults' prevalence estimates range from 0.5% to 13%.

Della Monica et al. [12] (p. 205), suggest that Michelangelo's David portraits phimosis: "*Regarding the genitalia, the testes appear proportionate and of apparent normal volume for an adult male* (G5 Tanner), *whereas the shaft corresponds more to that of an adolescent state* (G3 Tanner), *with a constriction of the glans as seen in phimosis.*" G3 and G5 refer to Tanner's [3] development stages of the male adolescent, based primarily on the penis size, as described in Appendix A.

When the penis exhibits torsion, hypospadias, or other congenital or acquired curvatures, accurately aiming the urinal central zone can pose a hazard. The true prevalence of hypospadias is difficult to estimate. Springer et al. [25] report substantial regional and ethnic variability across studies that screened a total of 90,255,200 births. Note that many reports provide contradictory prevalence data. A more recent study by Yu et al. [26] suggests that the overall prevalence of hypospadias may be increasing in several countries, although differences in data collection methods across programs represent a significant limitation of these findings.

Distal hypospadias (mild) includes glanular and subcoronal types, accounting for 60% to 70% of all cases. Midshaft hypospadias involves the penile shaft and accounts for roughly 20% of cases. Proximal hypospadias (severe) includes penoscrotal, scrotal, and perineal types. These are the most complex but least common, making up the remaining 5% to 10% of cases. All forms impair the fitness of the urine stream aim, and men with midshaft or proximal hypospadias prefer to use WC pans rather than standing urinals.

Some degree of penile curvature can cause the penis to bend upward, downward, or sideways, mainly when it is erect. In broader population-based surveys, up to 11.8% of men self-reported curvature, 48-49% ventral (downward), 24% lateral (sideways), 23% ventral and lateral (mixed), and 5% dorsal (upward).

Penis curvature can be congenital — Congenital Penis Curvature (CPC), or chordee — eventually in conjunction with hypospadias, abnormal foreskin or penile torsion. Prevalence is difficult to estimate, with reported incidences running from under 1% to as high as 10%, depending on the study and definition. Chung [27] 2018 population-based study in Australia found that 19% of men self-reported some degree of curvature, in general lower than 30^{o}. The wide range is likely due to many minor cases going unreported, but some studies estimate that the incidence of mild, isolated CPC is around 0.6%, see for instance Yachia et al. [28] and Chung et al. [27], see also https://my.clevelandclinic.org/health/diseases/15952-congenital-penile-curvature.

In view of Nabokov's fondness for puns, the title *Bend Sinister* of his 1947 second English novel [29] may be some endearing personal recollection. However, in the 1963 reedition, he explains that the title "*was an attempt to suggest a outline broken by refraction, a distortion in the mirror of being, a wrong turn taken by lif*e". In traditional heraldry, bend sinister is popularly associated with illegitimacy.

Non-congenital penis curvature can be caused by Peyronie's disease, a condition in which fibrous scar tissue forms in the deeper tissues under the skin of the penis. This causes curved, painful erections. The reported prevalence of Peyronie's disease varies widely in studies (from 0.3% to 20.3%) due to factors like age, ethnicity, and how the data was collected, and patients' embarrassment may be responsible for underestimation of the incidence (Schwarzer et al. [30]; Stuntz et al. [31]; Moghalu et al. [32], Paulis et al. [33]). In Sandean et al. [34] there is the curious statement: "*Oral therapy is highly desired by patients, but there are insufficient high-quality, adequately controlled, randomised studies to justify the routine use of any oral treatment for Peyronie disease. Most studies are small, short in duration, lack control groups, and use non-standardised treatment protocols. All such oral therapy use is off-label. Patients who demand oral treatment should be fully informed of the lack of sufficient credible evidence of efficacy*."

It is not clear whether this oral therapy is some form of conversation trying to convince the penis to not bend, or fellatio, easier than penetration for a twisted penis, or more prosaic swallowing pills.

4. Statistical Models for the Dimensions of Urinals Design

Owing to the Central Limit Theorem [35], there is a well-established preference in favour of the Gaussian law. The univariate Gaussian density appears to have influenced the classical design of urinal frontal rims, as illustrated in Figure 15, as well as the neo-baroque *Vineta* urinal and Duchamp's *Fountain* (Figure 3), whose frontal shapes resemble the central $[\mu-\sigma,\mu+\sigma]$ region of a univariate Gaussian distribution.

Figure 15 Univariate normal density function influencing the shape of urinals

Considering the characteristics of the human male anatomy, it is apparent that the frontal section — apparently based on the 68% central region [μ−σ,μ+σ] of the Gaussian distribution — is often too narrow, promoting urine spillage. For this reason, many modern urinal designs appear to draw inspiration from the bivariate Gaussian distribution, using concentration ellipses to shape the rim more effectively (see Figure 16).

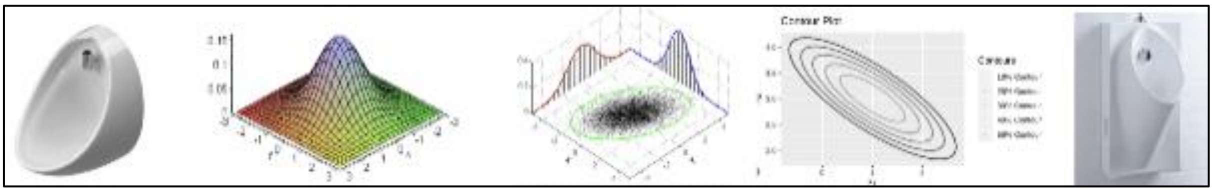


Figure 16 Bivariate normal density function and concentration ellipses, influencing the shape of urinals

This approach is more appropriate, as efficient aiming depends not only on the user's proximity and penis size (depth modelling) but also on the variability in direction and possible penile distortions (width modelling). Users also vary in height, particularly due to age, so extending the vertical dimension of the urinal is recommended to accommodate this variation. Since many urinals feature an almost flat vertical back, it is reasonable to assume that, in general, men direct their urine stream uniformly toward the back panel.

The distribution function of the standard Cauchy random variable $W_{1,1}$ is $F_{W_{1,1}}(w) = \frac{1}{2} + \frac{1}{\pi}\arctan(w), w \in R$. From the probability integral transform theorem, $\frac{1}{2} + \frac{1}{\pi}\arctan(W_{1,1})$ is the standard Uniform U, and therefore $\arctan(W_{1,1}) = \pi\left(U + \frac{1}{2}\right) \sim Uniform\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

The strict relationship $Y_{1,1} \sim \text{Uniform}\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \Leftrightarrow W_{1,1} = \tan(Y_{1,1}) \sim \text{Cauchy}(0,1)$ links the distance of the urine stream's impact on the back of the urinal relative to its centre with the angle of the penis relative to the vertical.

Consequently, it seems reasonable to assume that the Cauchy distribution — implying heavier tails — can be more appropriate for urinal width modelling, using an appropriate scale to accommodate the range of the penis possible directions during urination. Similar arguments hold in what concerns the height of the urinal. So, there are grounds to consider that a bivariate Cauchy model could be adequate, implying that the rim should contain a large enough, say approximately 0.68 (the approximate probability in [μ−σ,μ+σ] for a univariate Gaussian), probability concentration ellipse (see Figure 17). This aligns with the idea that urinals with a larger rim opening are more efficient in preventing urine from soiling the floor.

The Cauchy distribution is a Student-*t* distribution with one degree of freedom, which can be obtained, for example, as the quotient of two zero-mean Gaussian variables with equal variance. On the other hand, we can consider the Gaussian a Student with an infinite number of degrees of freedom, thus having maximum entropy.

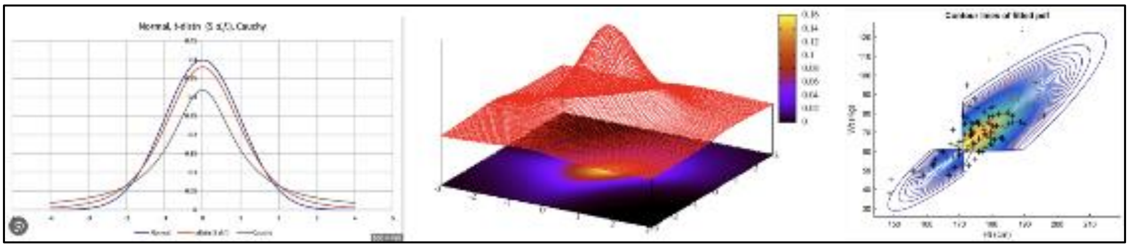


Figure 17 Comparing the tail-weight of normal, $t_{(5)}$ and Cauchy (left); Cauchy random pair and concentration ellipses (middle); contour lines, bivariate skew-*t* Mudholkar-Hutson model, source: Arnold et al. [36] (right).

The leftmost graphic in Figure 17 clearly shows that tail-weight decreases as the number of degrees of freedom increases. The asymmetries of testicles and penis position described in Section 3 also led us to consider, in the rightmost graphic of Figure 17, skewed Cauchy distributions, mixtures of half-Cauchy densities, and a bivariate version of the original Mudholkar-Hutson density, as investigated by Arnold et al. [36].

Note also that the recommendation to use Cauchy rather than Gaussian distribution results from considering that the hitting point in the back of the urinal is uniformly distributed. This is to a certain extent likely, due to the huge variability of human biometry and behaviour. In fact, a rationale of the type of using a flat non-informative prior in Bayesian Statistics.

The Uniform law is the simplest case $X_{1,1}$ of the $X_{\alpha,\beta}$ Beta family, $\alpha,\beta > 0$, with standard probability density functions $f_{X_{\alpha,\beta}}(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{K}\, I_{(0,1)}(x)$, shape parameters $\alpha, \beta > 0$.

The normalising constant in the denominator is $K = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\, \mathrm{d}x = B(\alpha,\beta)$, the value of Euler's Beta function for the pair of parameters $\alpha,\beta > 0$. Observe that $\alpha = \beta$ implies symmetry (main diagonal of Figure 18), $\alpha < \beta$ implies left asymmetry (graphs above the diagonal in Figure 18), and $\alpha > \beta$ implies right asymmetry (graphs below the diagonal in Figure 18).

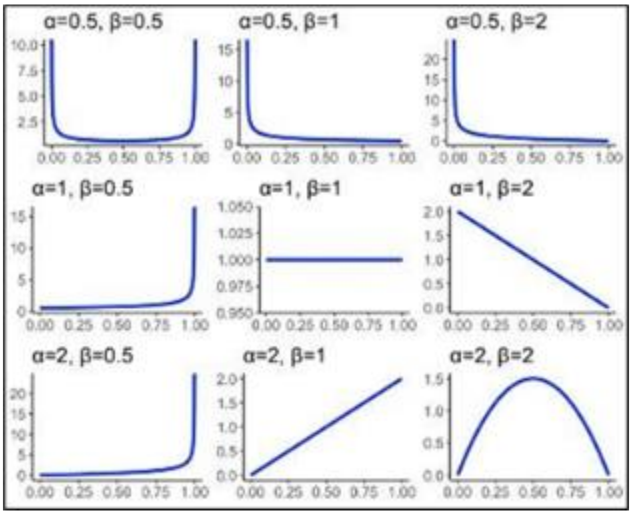


Figure 18 Some Beta densities shapes. The ArcSin density of $X_{0.5,0.5}$ concentrates on the endpoints of the support, the uniform $X_{1,1}$ is the equiprobability model, and the logistic parabola density of $X_{2,2}$ has mode 1.5 in the middle point

From herein we shall denote $X_{\alpha,\beta} \sim \text{Beta}(\alpha,\beta)$, with $\alpha,\beta > 0$, and using the location parameter $-\pi/2$ and scale parameter π, $Y_{\alpha,\beta} = \pi X_{\alpha,\beta} - \pi/2 \sim \text{Beta}(\alpha,\beta,-\pi/2,\pi)$, and the tanBeta random variable $W_{\alpha,\beta} = \tan(Y_{\alpha,\beta})$.

Inspired by the connection of the scaled and shifted $Y_{1,1} \sim \text{Uniform}\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ to the Cauchy random variable $W_{1,1} = \tan(Y_{1,1})$, we shall investigate simple tractable cases of the family of random variables $W_{\alpha,\beta} = \tan(Y_{\alpha,\beta})$, $\alpha,\beta > 0$.

Shifting the support to $[-\frac{\pi}{2},\frac{\pi}{2}]$, the $Y_{\alpha,\beta} = \pi X_{\alpha,\beta} - \frac{\pi}{2}$ random variable has probability density function

$$f_{Y_{\alpha,\beta}}(y) = \frac{\left(\frac{1}{2}+\frac{y}{\pi}\right)^{\alpha-1}\left(\frac{1}{2}-\frac{y}{\pi}\right)^{\beta-1}}{\pi\, B(\alpha,\beta)}\; I_{\left(-\frac{\pi}{2},\frac{\pi}{2}\right)}(y),$$

which when α= β simplifies to $f_{Y_{\alpha,\alpha}}(y) = \frac{\Gamma(2\alpha)}{\pi\,\Gamma^2(\alpha)}\left[\frac{1}{4}-\left(\frac{y}{\pi}\right)^2\right]^{\alpha-1}\; I_{\left(-\frac{\pi}{2},\frac{\pi}{2}\right)}(y)$.

Therefore, the random variable $W_{\alpha,\beta} = \tan(Y_{\alpha,\beta})$ has probability density function

$$f_{W_{\alpha,\beta}}(w) = \frac{1}{1+w^2}\,\frac{\left(\frac{1}{2}+\frac{\arctan(w)}{\pi}\right)^{\alpha-1}\left(\frac{1}{2}-\frac{\arctan(w)}{\pi}\right)^{\beta-1}}{\pi\, B(\alpha,\beta)}\; I_R(w).$$

In the symmetric case α = β, this expression simplifies to

$$f_{W_{\alpha,\alpha}}(w) = \frac{\Gamma(2\alpha)}{\pi\,\Gamma^2(\alpha)}\,\frac{1}{1+w^2}\left[\frac{1}{4}-\left(\frac{\arctan(w)}{\pi}\right)^2\right]^{\alpha-1}\; I_R(w).$$

Direct estimation of the shape parameters α and β is very difficult, but in view of the way $W_{\alpha,\beta} = \tan(Y_{\alpha,\beta})$ has been defined, $\frac{\arctan W_{\alpha,\beta}}{\pi}+\frac{1}{2} = X_{\alpha,\beta}$, and the shape parameters of $X_{\alpha,\beta} \sim Beta(\alpha,\beta)$ can be easily estimated by the method of moments, see the discussion in Ali et al. [37].

In the special case α = β = 2, the logistic parabola density $f_{X_{2,2}}(x) = 6\,x(1-x)\,I_{(0,1)}(x)$ (a name due to its close relation to logistic growth) is unimodal with mode $\left(\frac{1}{2},\frac{3}{2}\right)$, and stretching and relocating it to the support [−π/2,π/2], $f_{Y_{2,2}}(w) = \frac{6}{\pi}\left(\frac{1}{4}-\frac{y^2}{\pi^2}\right) I_{\left(-\frac{\pi}{2},\frac{\pi}{2}\right)}(y)$, and for $\left[-\frac{\pi}{2}\le y<\frac{\pi}{2}\right)$, the cumulative distribution function is $F_{Y_{2,2}}(y) = \frac{3\,y}{2\,\pi}-\frac{2\,y^3}{\pi^3}+\frac{1}{2}$.

From this, $f_{W_{2,2}}(w) = \frac{6\left[\frac{1}{4}-\left(\frac{\arctan(w)}{\pi}\right)^2\right]}{\pi(1+w^2)}\; I_R(w)$, and $F_{W_{2,2}}(w) = \frac{3\,\arctan(w)}{2\,\pi}-2\left(\frac{\arctan(w)}{\pi}\right)^3+\frac{1}{2}$.

Another tractable case is α = β = 1/2. The ArcSin density $f_{X_{\frac{1}{2},\frac{1}{2}}}(x) = \frac{1}{\pi\sqrt{x(1-x)}}\; I_{(0,1)}(x)$ has antimode $\left(\frac{1}{2},\frac{2}{\pi}\right)$.

For $Y_{\frac{1}{2},\frac{1}{2}} = \pi X_{\frac{1}{2},\frac{1}{2}}-\frac{\pi}{2}$ we get $f_{Y_{\frac{1}{2},\frac{1}{2}}}(y) = \frac{1}{\pi^2\sqrt{\frac{1}{4}-\left(\frac{y}{\pi}\right)^2}}\; I_{\left(-\frac{\pi}{2},\frac{\pi}{2}\right)}(y)$, and for $-\frac{\pi}{2}\le y<\frac{\pi}{2}$ the cumulative distribution function is $F_{Y_{\frac{1}{2},\frac{1}{2}}}(y) = \frac{2}{\pi}\arcsin\left(\sqrt{\frac{y}{\pi}+\frac{1}{2}}\right)$.

Therefore, the corresponding tanBeta random variable $W_{\frac{1}{2},\frac{1}{2}} = \tan\left(Y_{\frac{1}{2},\frac{1}{2}}\right)$ has probability density function $f_{W_{\frac{1}{2},\frac{1}{2}}}(w)\frac{\frac{1}{\pi^2(1+w^2)}}{\sqrt{\frac{1}{4}-\left(\frac{\arctan(w)}{\pi}\right)^2}}\; I_R(w)$, and cumulative distribution function

$$F_{W_{\frac{1}{2},\frac{1}{2}}}(w) = \frac{2}{\pi}\arcsin\sqrt{\frac{\arctan(w)}{\pi}+\frac{1}{2}}.$$

Note that the probability of ArcSin random variable $X_{\frac{1}{2},\frac{1}{2}}$ is concentrated in the endpoints of the support and, consequently, $W_{\frac{1}{2},\frac{1}{2}}$ has very heavy tails.

For comparison with the traditional Gaussian modelling, we shall also consider the probability density function of Y = arctan(Z):

$$f_Y(y) = \frac{1+\tan^2 y}{\sqrt{2\pi}}\exp\left(-\frac{\tan^2 y}{2}\right) I_{\left(-\frac{\pi}{2},\frac{\pi}{2}\right)}(y).$$

Figure 19 (left) shows the densities of the random variables $Y_{\frac{1}{2},\frac{1}{2}}$, $Y_{1,1}$ (Uniform), arctan(Z) and $Y_{2,2}$, where $Z \sim$ Gaussian(0,1), with common support $[-\pi/2,\pi/2]$. On the right, the figure shows the densities of their tangent transformations, $W_{\frac{1}{2},\frac{1}{2}}$, $W_{1,1}$ (Cauchy), Z and $W_{2,2}$, illustrating that the Gaussian distribution has lighter tails than the $W_{\frac{1}{2},\frac{1}{2}}$ and $W_{1,1}$ tanBeta densities, and that the tail weight of $W_{\alpha,\alpha}$ decreases as α increases.

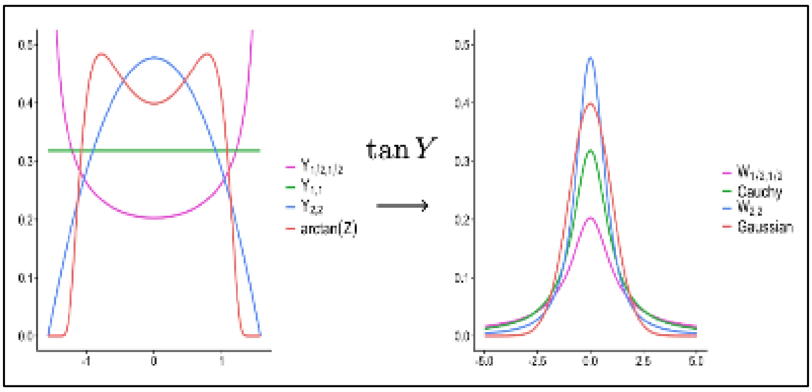


Figure 19 Densities of $Y_{\alpha,\alpha}$, $\alpha \in \{1/2, 1, 2\}$ and arctan(Z) (left), and of $W_{\alpha,\alpha} = \tan(Y_{\alpha,\alpha})$ and Z (right)

In what concerns bivariate modelling with those margins, Figure 20 compares the joint densities of bi-Gaussian and of the ones with marginal densities $W_{\frac{1}{2},\frac{1}{2}}$, Cauchy ($W_{1,1}$) and $W_{2,2}$.

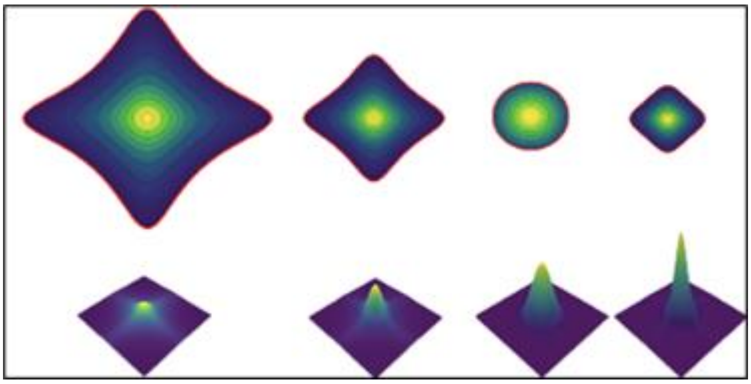


Figure 20 Comparison of 0.68 concentration contours (first line) and of bivariate densities (second line) with marginals $\boldsymbol{W_{\frac{1}{2},\frac{1}{2}}}$, $W_{1,1}$ (Cauchy), Gaussian and $W_{2,2}$

The random variables $W_{\frac{1}{2},\frac{1}{2}}$, $W_{1,1}$ (Cauchy) and $W_{2,2}$ have heavier tails, larger than the Gaussian (in Figure 19, right). The density of $W_{2,2}$ is more peaked than the Gaussian density, with slightly lighter tails. In the first line of Figure 20, the 0.68 probability contours of the standard bi-Gaussian and bi-$W_{2,2}$ are approximately of the same size, but the 0.68 probability contours of the standard bi-Cauchy and bi-$W_{\frac{1}{2},\frac{1}{2}}$ are, as expected, much larger.

Note that the mathematical presentation uses the simple standard forms $W_{\alpha,\beta}$, that should be considered representatives of their Khinchine's type $\lambda + \delta W_{\alpha,\beta}$, *where* $\lambda \in R$ is a location parameter and $\delta > 0$ is a dispersion parameter. It would obviously be very unrealistic to think that the range $R = \pi/2-(-\pi/2) = \pi$ is the range of the urination hitting point in the back of the urinal. While it is plausible to consider that an admissible location shift is 0, scale parameters γ_H and γ_V in what regards horizontal and vertical scatter, respectively, must be incorporated in the model. There are no reliable statistics for this, but a conservative opinion accepts that the horizontal range $\gamma_H\pi$ = 15 cm. In what regards the vertical range, as the height of users implies that instead of a central point, we must consider that the aiming point is variable, an acceptable modelling asset is $\gamma_V\pi$= 20 cm.

Further, bivariate Cauchy ($\gamma_H W_{1,1}, \gamma_V W_{1,1}$) contours for modelling the rim of the urinal would be appropriate assuming uniformity rules for the urination aiming point. On the other hand, if there are reasons to consider that users behave trying as much as possible to avoid spilling (for instance, if there is a marked target), ($\gamma_H W_{2,2}, \gamma_V W_{2,2}$) would be plausible. In case users don't behave neatly, either caused by some anatomic propensity, or due to drunken haze, or simply resulting from bad manners, ($\gamma_H W_{\frac{1}{2},\frac{1}{2}}, \gamma_V W_{\frac{1}{2},\frac{1}{2}}$) would be more appropriate. Obviously, mixed models in what concerns the horizontal and the vertical scatter can be more realistic.

5. Discussion

There are sound grounds to believe that scrotal asymmetry influences some asymmetry in the direction of the human male urine stream but, from aesthetic considerations, urinals must continue to be symmetrical.

As plainly stated in Section 3, statistics of the prevalence of penile abnormalities are very unreliable, varying widely in studies. Most studies use convenience samples, and even those using random samples are biased due to patient's embarrassment, causing problematic missing data. As far as we know, no random-response techniques, adequate to circumvent cringeworthy questions, have been used in interviews. In what regards the penis dimensions (flaccid and erect), the meta-analysis findings of Veale et al. [21], indicate smaller values than studies including self-reported measurements. This may result from the eligibility criteria, namely the exclusion of samples with a congenital or acquired penile abnormality, and the use of 20,000 simulated Gaussian pseudo-observations to generate nomograms of penis size. More recent meta-analyses (Belladelli et al. [22], Mostafaei et al. [23]) also found average lengths smaller than average overestimates in studies using self-reported measurements, but as the discussion in Section 3 suggests, clinician measurements can result in underestimation.

Gathering of reliable data on the male anatomy and behaviour would provide further insight and guidance for the efficient and ergonomic design of hygienic urinals, namely statistics of the preferred distances from the urinal and of penises lengths to determine ideal urinal depth, and statistics of the penis deviation angle from the central vertical position and of congenital or acquired curvature for guidance on urinals width. A comprehensive *Urinal Behaviour in the Human Male* to match Kinsey et al. [38] *Sexual Behavior in the Human Male* would be welcome but, with the actual trends of research, a deep investigation of the matter seems unlikely to appear in the short term. Chapter 2 ("Interviewing") of Kinsey et al. [38] offers very practical guidance on easing human contact. The critical report by Cochran et al. [39], commissioned by the American Statistical Association to assess Kinsey et al. [38] statistical methods, is a superb argumentative study on the applications of Statistics.

Urinals with wider rims would more efficiently prevent urine from spilling onto the floor. Some modern stylish designs, as the *Drop* urinal in Figure 2, are aesthetically beautiful, but inconveniently slim. To a certain extent, the inclusion of a target in the urinal (for instance a blowfly picture) can minimize inadequate urine stream direction but, from aesthetic considerations, this doesn't seem appropriate in most luxury designs depicted in Section 6.

The urinal in Sodoma bar (Reykjavik) (Figure 14, right) suggests that a visual target in the urinal motivates users to aim at it. Therefore, the drawings of the bivariate Gaussian contour depicted in Figure 20 can be used in thin and/or flat urinals, as illustrated in Figure 21 (right), to diminish spilling to the floor.

Figure 21 Flat urinal with a hitting target

The Gaussian density shape seems to be a sound background normal guidance for the design of urinals. At first sight, considering a [μ−2σ,μ+ 2σ] > 95% probability interval could seem appropriate, but the leftmost graphic in Figure 17 clearly shows that its relative peakedness, compared to other bell-shaped, heavier-tailed alternatives, would not effectively prevent urine from spilling onto the floor. Therefore, bivariate bell-shaped models would offer more adequate guidance, even when the statistical background contour is not immediately visible, as in the luxury urinals shown in Section 6.

As explained in Section 4, a bivariate Gaussian distribution might be better replaced by heavier-tailed alternatives, unless some target mark near the centre of the urinal acts as an efficient decoy guiding the urination stream aim. If an aiming target exists, a light-tailed contour, based for instance on $W_{2,2}$, justifies the use of a slim design.

To maintain public decency, modern street open-view urinals (Figure 6, right) feature very narrow openings, which encourage users to lean in closely and, in principle, help prevent urine spillage. However, as previously discussed, some non-open-view stylish designs also feature a narrow rim, which can either impair floor hygiene, or force the user to urinate from very close range, eventually originating discomfort and concern about disease transmission. Moreover, thorough cleaning of the basin of narrow rim urinals, as the anti-spilling urinals in the middle of Figure2, the *Groove* urinal (Figure 28) or the *Pufferfish* urinal (Figure 33) seems an impossible mission.

It therefore appears advisable, at the design stage, to consider adjusting the urinal width, as illustrated in Figure 22 with the *Drop* urinal. The inclusion of a target — preferably shaped like the contours in Figure 20 — can promote more appropriate user behaviour. In this context, it is recommended that the urinal width be approximately twice the width of the target.

In fact, augmenting the rim of the urinal to contain a predetermined concentration contour is easily feasible using the 'Adjust size' features of *Preview*.

Quantiles Z_p of the standard Gaussian(0,1) distribution are tabulated, or easily obtainable in a spreadsheet. Recalling that $Z_{0.8413} - Z_{0.1587} = 2$, in Table 4 we compute the width of the central $[\chi_{0.1587}, \chi_{0.8413}]$ interval of $W_{\frac{1}{2},\frac{1}{2}}$, $W_{1,1}$ and $W_{2,2}$. For that end, the quantile function of $W_{\frac{1}{2},\frac{1}{2}}$ is

$$\chi_p = \tan\left[\pi\left(\sin^2\left(\frac{p\pi}{2}\right) - \frac{1}{2}\right)\right],$$

and the quantile function of the Cauchy $W_{1,1}$ random variable is

$$\chi_p = \tan\left[\pi\left(\sin^2\left(\frac{p\pi}{2}\right) - \frac{1}{2}\right)\right].$$

The numerical solution of

$$\chi_p : \frac{3\arctan(\chi_p)}{2\pi} - 2\left(\frac{\arctan(\chi_p)}{\pi}\right)^3 + \frac{1}{2} = p$$

gives approximate values of the quantiles of the tanBeta(2,2) $W_{2,2}$ random variable.

In what concerns the $W_{2,2}$ random variable, we can compute numerically the quantiles, obtaining $\chi_{0.8413} - \chi_{0.1587} = 1.7356$.

Table 4 Width of the central 0.68 probability interval

Distribution	Quantile function χ_p	$\chi_{0.8413}$	$\chi_{0.8413} - \chi_{0.1587}$
tanBeta$\left(\frac{1}{2},\frac{1}{2}\right)$	$\tan\left[\pi\left(\sin^2\left(\frac{p\pi}{2}\right) - \frac{1}{2}\right)\right]$	5.1657	10.3315
tanBeta(1,1) — Cauchy	$\tan\left[\pi\left(p - \frac{1}{2}\right)\right]$	1.8367	3.6734
Gaussian(0,1)	$\Phi^{-1}(p)$	1.0000	2.0000

tanBeta(2,2)	$\chi_p: \frac{3\arctan(\chi_p)}{2\pi} - 2\left(\frac{\arctan(\chi_p)}{\pi}\right)^3 + \frac{1}{2} = p$	0.8678	1.7356

It is well known that P[-1<Z<1]=0.68; the width of the 0.68 probability central of the tanBeta $W_{\frac{1}{2},\frac{1}{2}}$ (10.3315), $W_{1,1}$ (3.6734) and $W_{2,2}$ (1.7356), and so the diameter of the 0.68 probability concentration contours bi-$W_{0.5,0.5}$, bi-$W_{1,1}$ and bi-$W_{2,2}$ are approximately 5.12, 1.83 and 0.87, respectively, wider than the 0.68 contour of the bi-Gaussian. These values can be used at the design stage for comparing results of resizing the width, to evaluate whether the alteration is welcome — since obviously aesthetic and ergonomist principles must have precedence. For example, with the *Drop* urinal, Figure 22 shows that the Cauchy choice seems a good compromise of beauty and functionality. $W_{\frac{1}{2},\frac{1}{2}}$ is obviously the safest choice but fails to have the stylish aspect of the other options. $W_{2,2}$, although elegant, is indeed too thin to be compatible with tidiness (the actual Gaussian shape itself seems too thin).

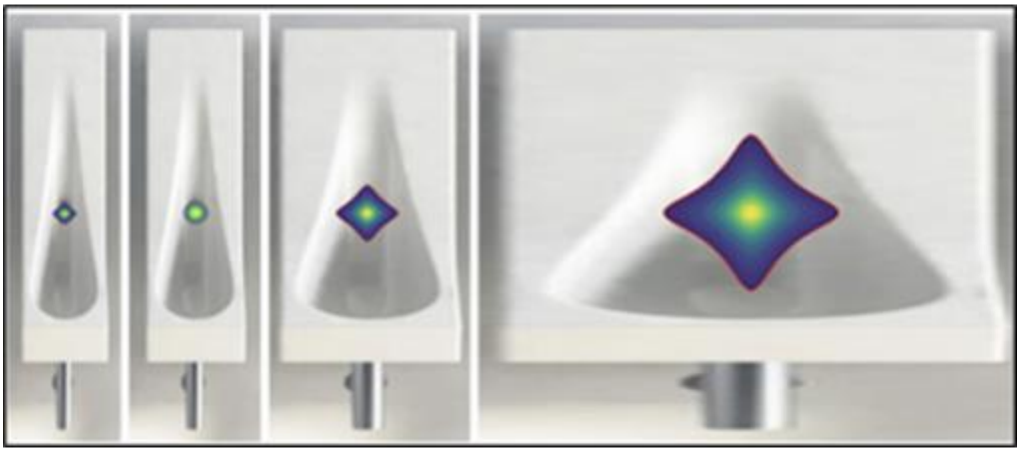


Figure 22 Resizing the *Hidra Drop* urinal (from left to right: $W_{2,2}$, Gaussian, Cauchy, $W_{0.5,0.5}$)

The splash-free urinals *Cornucopia* and *Nautilus* point at new objectives for the design of urinals, since it is claimed that they reduce urine splashback to 1.4% of what traditional commercial designs produce by maintaining the impinging angle below the critical value, for technical details cf. Thurairajah et al. [40]. As a result, aside from reducing bacteria and ammonia levels. Further, savings in cleaning materials, namely on the very valuable water resources, are enormous.

New, powerful tools for industrial design, such as AI-powered photo editors like *Pixlr Express* (https://pixlr.com/express/), enable the creation of more interesting and sophisticated results. For example, in Figure 23, we show a simple alteration of the rim of the left tenor horn urinal.

Figure 23 *Pixlr Express* alteration of the rim of the leftmost tenor horn urinal from a bar in Freiburg

By considering different perspectives, evaluating the available evidence, and minimising biases, a critical assessment of the expected behaviour of urinal users can inform the final design choice. Experience indicates that, in what concerns the usual commercial urinals, increasing dimensions, namely the width, would contribute to reduce dirtying the floor. Common sense seems to dictate simply to augment the scale of traditional urinals. The present study, and particularly the statistical modelling in Section 4, furnish scientific basis to decide on the increasing factor that should be used.

In locations or situations where users are likely to use urinals carelessly, it is advisable to select urinals with a broad opening or rim, potentially considering at least the bi- $W_{\frac{1}{2},\frac{1}{2}}$ contours. This is what happens for instance in pubs, where loaded clients may need to alleviate the ale excess intake, or of other kinds of beer. In this situation, several men urinate with a “sprinkler effect”, i.e. aiming at an imaginary left-right and up-and-down trajectory instead of at a fixed point. For that reason, several breweries and pubs patrons choose to recycle huge beer kegs as urinals (see Figure 24).

Figure 24 Beer kegs recycled as urinals

6. Gallery of Art and Design Luxury Urinals

Some stylish luxury urinals appeared in the nineteenth century, with elaborated ornamental decoration, as the neo-baroque 1895 *Vineta Urinoir* depicted in Figure 3.

The 1917 *Fountain* urinal, by Marcel Duchamp (Fig, 3), is regarded by art historians as a major landmark in twentieth century art. Duchamp stated that readymade sculptures are “*everyday objects raised to the dignity of a work of art by the artist’s act of choice*.” In Duchamp’s presentation, the urinal’s orientation was altered from its usual positioning.

To a certain extent, a similar intention of dignifying functional devices transforming them in artistic sculptures appears in Clark Sorensen’s statement “*My work reflects my interest in combining beauty with function and exploiting the discord between opposing elements in life and in art. The contradiction of taking an unsightly urinal and transforming it into a graceful object like a flower or shell is a potent combination. Because most people think of a urinal as being ugly or dirty, it is the perfect object to beautify*. (https://www.clarkmade.com/about.htm).

Sorensen’s urinal sculptures, see Figure 25, are beautifully realistic magnifying versions of perfect exemplars of flowers and shells whose shape is appropriate to fulfil the function, with a rim adequate for aiming the urination stream.

Figure 25 Clark Sorensen flower and shell urinal sculptures, courtesy of the artist

The Venus flytrap (*Dionaea muscipula*) and the crimson pitcher plant (*Sarracenia leucophylla*) are carnivorous plants, happily the marvellous Sorensen's urinals (Figure 26) cannot react as their nature inspirations…

Figure 26 Clark Sorensen Venus flytrap and pitcher plant urinals

In https://www.clarkmade.com/index.htm the reader can watch other beautiful Clark Sorensen sculptures, namely several orchids, that are endearing flowers, with a nice receptacle. It is interesting to recall that their name originated by the testicle ('ορχις) aspect of their roots (Figure 27).

Figure 27 Orchid testicle shaped roots

The design of urinals is a flourishing industry, producing very stylish results (Figure 28). However very slim rims can be inadequate in what concerns the hygiene of the floor.

Figure 28 Designs for *Drop* and *Origami.* Fabrizio Batoni Artwork *Groove* and Philip Watts *Spoon* urinals

For those who appreciate the usage of urinating in the washbasin, interesting modern ergonomic designs encourage users to wash their hands after urinating, as the elegant washbasin in Figure 29 (left), or what some experts consider to be a combo washbasin/urinal (Figure 29 middle).

Alternatively, as users may have very different legs lengths, enthusiasts of washbasin peeing may consider the Johan Kauppi and Lars Sundström's *Abisco* notable washbasin (Figure 29, right) a welcome design for a modern trough urinal.

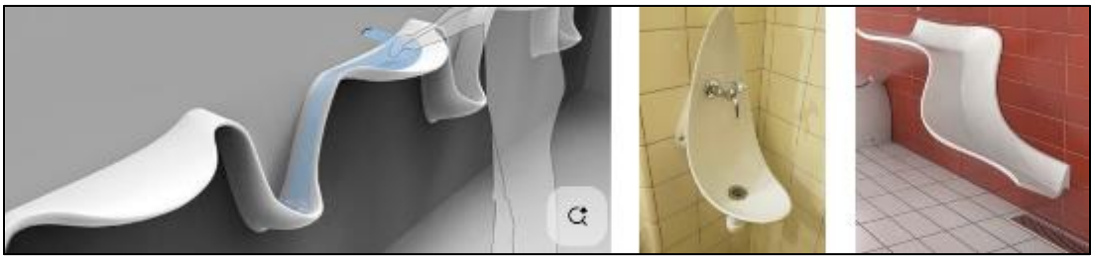


Figure 29 Combo washbasin/urinals and Johan Kauppi and Lars Sundström's *Abisco* washbasin

Forcing urinating during an erection is difficult due to the prostate constriction of the inner part of the urethra, and therefore urinals provoking some degree of sexual enticement (see Figure 30 and 31) can cause discomfort. On the other hand, menacing shapes, such as the guillotine (Figure 31) or the shark (Figure 33) urinals presumably contribute to maintain a flaccid penis, thus facilitating comfortable urination. And, as Mme de Pompadour promptly replied when asked whether she preferred love making *avant* (before), *pendant* (during) *ou après* (after) the act: "*Moi, j'aime avant, parce qu'après c'est pendant*".

Minissale [39] analyses sexual motivations in many urinals design. In fact, sexual enticement, using voyeurism, in Sofitel Queenstown, New Zealand, designed by Mark Perriam, Brett Tailor and Cam Marsh (Figure 30), or chaste female nun bodies, produced some very appealing urinals (Figure 3, middle).

Figure 30 Voyeurism suggestion in the men's restroom in Sofitel Queenstown, New Zealand

On the other hand, some urinals explicitly representing readiness to copulation and other sexual, eventually sadomasochism practices, namely the menace of guillotine castration (Rheinfels Castle, Germany), as shown in Figure 31, can be considered offensive.

Figure 31 Provocative/offensive, eventually sadomasochism, urinals

Training and maturation of the prostate contribute to the control of infantile incontinence, and in adolescents and young adults, urination typically produces a strong, nearly ideal parabolic stream (see Figure 31).

Figure 32 Healthy prostate parabolic urination stream ("*Joey peeing*" photo by Matthew Romack)

With age, prostate changes can lead to urinary difficulties, including secondary incontinence and urine splashing or spilling. The Brussels Mannekenpis, though the replica on exhibition is now 60 years old, remains youthful, and the photo in Figure 31 (left) shows that he still maintains a clear, strong parabolic urine stream.

While young men can urinate from a distance, as depicted in "*Joey peeing*" photo (Figure 31, right), mature men often need to stand closer to the target — especially if the receptacle is small, as in the infamous Mapplethorpe photographs "*Jim and Tom, Sausalito*" (1977), https://www.tumblr.com/categorized-art-collection/59014549792/robert-mapplethorpe-jim-and-tom-sausalito .

This, together with Salvador Dalí's *Mae West Lips* sofa, may have inspired the mouth-shaped *Kisses Urinal*, designed in 2000 by Meike van Schijndel (Figure 32), that surely has a sensual enticement purpose. The gothic mouth shaped urinal in Mini Bottle Gallery Museum, Oslo (Figure 32), with lateral mirrors, is obviously ergonomic and appealing for those who, having perhaps a voluminous belly, need to use a peniscope.

Figure 33 Salvador Dalí's *Mae West Lips* sofa and the Meike van Schijndel's *Kisses urinal*. Mouth shaped Gothic urinal with lateral mirrors in Mini Bottle Gallery Museum, Oslo

Other mouth shaped urinals (Figure 33) use frightening teeth, eventually being shark shaped. On the other hand (or, in this case, mouth), the very narrow mouth opening of the *Pufferfish* urinal almost forces the user to introduce the penis in it, with potential diseases contamination, as already commented for other narrow rim urinals. Political leaders may also be an inspiration for mouth shaped urinals, as Clark Sorensen's "*The Presidential Urinal*".

Figure 34 Mouth urinals: Shark mouth, *Pufferfish* and Clark Sorensen's *Presidential* urinals.

Special gents deserve special urinals. For concert halls, an obvious luxury recommendation is the use of musical instruments, as the recycled tenor horn urinals made by Martin Hartmann to a bar in Freiburg, shown in Figure 23 (left). The weird coffin urinals depicted in Figure 34 (left) seem fit for mortuary parlours or for cemeteries. The penis shaped *Capri* urinals, as depicted in Figure 34 (middle), may be an interesting choice for instance to gay bars. As Minissale [41] states, in relation to the third urinal in Figure 30, "*Of course, in a gay bar, this phenomenology would change, reminding us that context changes signs and deflects their meanings, as well as disturbs heterosexual universalizing phenomenology*."

Figure 35 Urinals for mortuaries, for bars, and for economists/statisticians/brokers associations: coffin and *Capri* urinals; Clark Sorensen *Down the Drain* urinal

Economists, statisticians or brokers associations deserve time series decorated urinals, as the Sorensen's "*Down the Drain*" urinal, Figure 34 (right), although for statisticians we would prefer more optimistic graphs.

The *Joe lounge chair* inspired in Joe DiMaggio baseball glove, designed by De Pas, D'Urbino and Lomazzi in 1970, plainly shows that the unexpected in Design can produce superb results. Creative designers can use many natural or manmade devices as inspiration to create original urinals for special places or people (Figure 35) — pumpkins for Halloween addicts, balls for football stadiums, Sherlock Holmes pipe for tobacco companies and private eye agencies, skulls for tattoo shops, amethyst geodes for wellness practitioners…

Figure 36 Inspiring natural and manmade devices

A quote often attributed to Einstein, "*Two things are infinite: the universe and human stupidity; and I'm not sure about the universe*," should include also creative imagination as a third possibly infinite thing.

More inspiration can be gathered in a virtual visit to the *Museo del Orinal* (Urinal Museum) in Ciudad Rodrigo, Spain, https://turismo.ciudadrodrigo.es/museo-del-orinal/ .Note however that "orinal" refers mainly to chamber pots, not to the urinals considered in this study.

6.1. Tanner's Male Adolescent Genitalia Development Stages

According to Tanner [3], the stages for the genitalia, illustrated in Figure 36, are as follows:

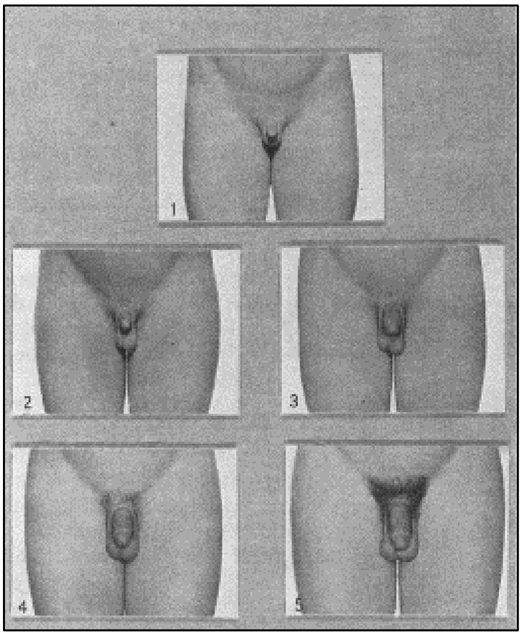


Figure 37 Tanner's G1–G5 male adolescent genitalia development stages (source: Tanner [3])

- *Stage 1:* Pre-adolescent. Testes, scrotum and penis are of about the same size and proportion as in early childhood.
- *Stage 2:* The scrotum and testes have enlarged and there is a change in the texture of the scrotal skin. There is also some reddening of the scrotal skin, but this cannot be detected on black and white photographs.
- *Stage 3:* Growth of the penis has occurred, at first mainly in length, but with some increase in breath. There has been further growth of testes and scrotum.
- *Stage 4:* Penis further enlarged in length and breadth, with development of glands, testes, and scrotum further enlarged. there is also further darkening of the scrotal skin, but this is difficult to detect on photographs.
- *Stage 5*: Genitalia adult in size and shape. No further enlargement takes place after stage 5 is reached.

7. Conclusion

Despite the human male anatomical variability, most urinal designs seem to take for granted that the penis is straight and vertically positioned, and that the urine stream is directed to the centre of the urinal opening. These assumptions are baseless and unreliable. Taken together, this study considerations support the common-sense conclusion that urinals should be designed with a broader target zone to reduce the likelihood of floor dirtying, further indicating enlarging factors based on scientific statistical modelling.

Compliance with ethical standards

Acknowledgments

We are grateful to Matteo Della Monica and co-authors for permission to reproduce text and Figure 9 (left) from [12]; to David Veale, Sarah Miles, Sally Bramley, Gordon Muir, and John Hodsoll for permission to reproduce the nomograms in Figure 12 from [21]; and to Barry C. Arnold, Héctor W. Gómez, Héctor Varela, and Ignacio Vidal for permission to reproduce Figure 17 (right) from [36]. Gregory Minissale kindly permitted us to use text and images from [39]. The quotation from the book by Sandean et al. [34], appearing at the end of Section 3, is reproduced under the terms of the Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International licence (CC BY- NC-ND 4.0). In Appendix A, "Tanner's Development Stages" is reproduced from Tanner (1969), with kind permission from Elsevier. We also thank Clark Sorensen for permission to reproduce photographs of his remarkable sculptures of flowers and shell-shaped urinals (Figures 3 (right), 25, 26, 34 and 35), and Meike van Schijndel for her photograph of the *Kisses urinal*. Figure 33. Figure 7 (right) appears to be a remix or collage of eight heads combined with a promotional photograph from the film *Solo: A Star Wars Story*. Figure 1 (left), the Doryphoros in Figure 7, Figure 14 (right), Figures 21, 29, 30, and the "*Joey peeing*" photograph by Matthew Romack in Figure 32 (right) are reproduced under the terms of the

Creative Commons Attribution–ShareAlike 4.0 International licence. Finally, we thank the many anonymous contributors who shared such striking images online, thereby enriching the illustrative material of this paper.

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No conflict of interest to be disclosed.

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