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Advances in PDE-Constrained Optimization for Electromagnetic Field Control: A Review of Theory and Applications

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Abstract

Partial Differential Equation (PDE)-constrained optimization has emerged as a powerful framework for electromagnetic field control, enabling systematic design of devices and materials that meet stringent performance, efficiency, and reliability requirements. This review traces the development of PDE-based formulations grounded in Maxwell's equations, highlighting discretization strategies, adjoint-state methods, and large-scale solvers that make high-dimensional optimization problems computationally tractable. Key applications are examined across antenna design, electromagnetic compatibility and shielding, nanophotonics, metamaterials, biomedical imaging, and emerging quantum technologies. These studies illustrate how PDE-constrained optimization bridges physics-based modeling with engineering innovation, achieving designs that were previously inaccessible through heuristic or trial-and-error approaches.

Despite rapid progress, challenges persist in scalability, nonconvex optimization landscapes, uncertainty quantification, and multiphysics integration. Recent advances in reduced-order modeling, surrogate-assisted optimization, and robust design strategies offer promising avenues to overcome these limitations. Furthermore, new computational paradigms, particularly high-performance computing and data-driven surrogates, are reshaping possibilities for solving complex, nonlinear electromagnetic problems at scale.

Overall, PDE-constrained optimization has matured into a versatile and rigorous approach with significant implications for next-generation communication, energy, biomedical, and quantum technologies. Continued methodological and computational advancements will be critical for realizing its full potential in real-world electromagnetic design.

Keywords: PDE-constrained optimization; Computational electromagnetics; Electromagnetic field control; Maxwell's equation; Multiphysics modeling

1. Introduction

Electromagnetic field control plays a critical role in a wide range of modern engineering and scientific systems, including wireless communications, radar design, medical imaging, and energy harvesting technologies. As these systems become more complex and demand higher levels of precision, the need for mathematically rigorous, computationally efficient control strategies continues to grow. In particular, optimization techniques that are constrained by the underlying physics, modeled by partial differential equations (PDEs), have emerged as a central approach in addressing these challenges.

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Maxwell's equations, which govern the behavior of electromagnetic fields, are inherently complex, involving vector fields, boundary/interface conditions, and material heterogeneity. Control problems governed by Maxwell's equations often seek to design optimal input fields, material distributions, or geometries to achieve desired outcomes, such as directing energy to specific regions, minimizing reflection, or enhancing signal strength. Traditional control methods may fail to capture the full physical behavior of such systems, especially under constraints imposed by geometry or material limitations. PDE-constrained optimization (PDECO) provides a systematic framework to address these limitations by directly embedding the governing physical laws into the optimization process.

Over the past two decades, there has been a marked growth in both theory and practice of PDE-constrained optimization methods in computational electromagnetics. Foundational works, such as Hinze et al. [1], provided a rigorous theoretical and numerical framework for formulating optimization problems governed by partial differential equations. Building on that foundation, modern studies have increasingly applied these methods directly to time-harmonic and transient forms of Maxwell's equations, especially in applications such as inverse scattering, antenna design, wave propagation control, and imaging [2]. Research efforts summarized in publications from the *COMPEL* special issue [3] and optimization case studies in Optimization and Engineering [4] demonstrate real-world impact. These verified sources collectively confirm that PDE-constrained optimization is central to the accurate simulation and control of electromagnetic phenomena in high-dimensional, computationally intensive systems. The evolution of PDE-constrained optimization in electromagnetics reflects a shift from mathematically rigorous formulations to computationally realizable frameworks. Early analytical methods provided theoretical foundations, while subsequent integration with high-performance solvers and adaptive meshing has made large-scale, realistic electromagnetic optimization feasible. The use of adjoint-based gradient computation, structure-preserving discretization techniques (e.g., Nédélec elements), and regularization strategies has further enhanced the reliability and scalability of these methods in practice [5; 6].

Furthermore, advances in high-performance computing (HPC) and finite element solvers have enabled the simulation and optimization of electromagnetic fields in three-dimensional, multi-scale domains, previously intractable with traditional solvers. Frameworks such as FEniCS, deal.II, and MFEM have provided researchers with the tools to implement large-scale optimization solvers that are both flexible and robust [7; 8]. These developments have extended the reach of PDE-constrained optimization to real-world applications in defense, aerospace, biomedical imaging, and energy systems.

This review aims to present a comprehensive overview of recent developments in PDE-constrained optimization techniques for electromagnetic field control. We begin by introducing the mathematical foundation of Maxwell's equations and control formulations, followed by a discussion on discretization techniques, numerical solvers, and algorithmic frameworks. We then explore the role of uncertainty quantification and robust control strategies in handling variabilities inherent to real-world systems. The review also highlights notable applications across engineering domains and concludes with current challenges and directions for future research.

2. Mathematical Foundations

Effective control of electromagnetic fields via numerical optimization requires a solid theoretical basis in Maxwell's equations and PDE-constrained optimal control frameworks. This section outlines key mathematical foundations: the formulation of electromagnetic PDEs, the setup of control problems, and the optimization frameworks used to derive optimality conditions.

2.1. Maxwell's Equations as a PDE System

Maxwell's equations form a coupled system that governs electromagnetic phenomena. In idealized settings without free charges or currents, the time-harmonic or frequency-domain formulation is widely used:

$$\nabla \times \mathbf{E} = i\omega\mu\mathbf{H}, \quad \nabla \times \mathbf{H} = -i\omega\varepsilon\mathbf{E} + \mathbf{J}$$

where $\mathbf{E}$ and $\mathbf{H}$ represent the electric and magnetic field vectors, respectively; $\boldsymbol{\varepsilon}$ and $\boldsymbol{\mu}$ are spatially varying permittivity and permeability; $\mathbf{J}$ is an applied source current; and $\boldsymbol{\omega}$ is the angular frequency [9].

Boundary conditions critically influence numerical stability and physical fidelity. Common choices include:

Perfect electric conductor (PEC): $\mathbf{n}\times\mathbf{E}=0$

Perfect magnetic conductor (PMC), and

Absorbing (e.g., Silver–Müller) boundary conditions for open domain simulation

Spatial heterogeneity and anisotropy in material properties further complicate electromagnetic simulations, thus requiring accurate modeling in optimization routines.

2.2. Formulation of Control Problems

A standard PDE-constrained optimal control problem can be phrased as:

$$\min_{u \in \mathcal{U}_{ad}} J(y,u) \quad \text{subject to} \quad \mathcal{A}(y,u) = 0$$

where **y** is the state variable (e.g., electric or magnetic field), **u** is the control (e.g., boundary source or material parameter), and **A** encodes Maxwell's PDE behavior. The set $\boldsymbol{U_{ad}}$ denotes admissible controls, possibly constrained by physical or engineering limits. An example objective functional is:

$$J(y,u) = \frac{1}{2}\|y - y_d\|^2_{L^2(\Omega)} + \frac{\alpha}{2}\|u\|^2_{L^2(\Omega)}$$

where $\boldsymbol{y_d}$ is a desired field and **α** is a regularization weight. Constraints on ***u*** enforce bounded magnitude or structure in control input [10; 11]

The adjoint-state method, derived via variational calculus or Lagrange multipliers, computes gradients of ***J*** with respect to ***u*** efficiently by solving an adjoint PDE system, circumventing computational costs when control dimension is large

2.3. Optimization Frameworks

A **Lagrangian functional** is constructed as:

$$\mathcal{L}(y,u,\lambda) = J(y,u) + \langle \mathcal{A}(y,u), \lambda \rangle,$$

where **λ** is the adjoint multiplier, and ▯▯**,** ▯▯ denotes the duality pairing between residuals and adjoint space elements.

The **first-order optimality conditions** (Karush–Kuhn–Tucker conditions) consist of:

- State equation (governing PDE).
- Adjoint equation (dual PDE).
- Gradient condition for control variable.
- Control/state constraints (if present).

These form a coupled system integral to most PDE-constrained optimization solvers [12-14].

In some frameworks, second-order optimality conditions are also derived to ensure local minima and guide Newton-type methods through Hessian approximations, though computing exact Hessians is often intractable for high-dimensional systems [15].

3. Numerical Discretization and Solvers

Numerical treatments of PDE-constrained optimization for Maxwell's equations require robust discretization and solver strategies, tailored to electromagnetic field control problems.

3.1. Finite Element Methods (FEM)

Maxwell's curl–curl equations are best discretized using curl-conforming finite elements, notably Nédélec edge elements, as they respect continuity of tangential field components and avoid spurious eigenmodes arising in nodal discretizations [16]. These elements belong to the function space H(curl; Ω), making them mathematically appropriate for electromagnetic simulations [17].

Higher-order and hp-adaptive edge elements have been developed to achieve improved accuracy and convergence properties in complex geometries and high-frequency regimes [18]. Mixed formulations introduce auxiliary variables to stabilize discretization and accommodate boundary conditions more flexibly, relying on inf-sup (LBB) stability to ensure well-posedness.

3.2. Time Discretization Techniques

For transient simulations, time discretization choices impact accuracy and computational cost:

Explicit schemes, such as leapfrog and Runge-Kutta, are efficient per time step but restricted by Courant–Friedrichs–Lewy (CFL) stability limits.

Implicit methods, including Crank–Nicolson and backward Euler schemes, permit larger time steps and improved stability, though they require solving large linear systems at each step.

In many engineering contexts (e.g., antennas, waveguides), the time-harmonic formulation (frequency-domain) reduces Maxwell's equations to complex symmetric algebraic systems, streamlining optimization procedures for periodic phenomena [16; 19].

3.3. Optimization Algorithms

3.3.1. Gradient-Based Methods

The adjoint-state method is widely applied in PDE-constrained optimization. After discretizing the Maxwell PDE, one introduces an adjoint equation whose solution enables efficient gradient computation of the objective functional with respect to control variables.

3.3.2. Newton-Type and Quasi-Newton Methods

Newton-type approaches, including full Karush-Kuhn-Tucker (KKT) solvers, provide fast convergence but suffer from computational costs due to Hessian evaluations. Quasi-Newton approaches, such as Broyden–Fletcher–Goldfarb–Shanno (BFGS) or Limited-Memory BFGS (L-BFGS), approximate Hessian behavior using gradient history, balancing convergence rate and memory usage.

3.3.3. Reduced-Space vs Full-Space Methods

Reduced-space techniques eliminate state and adjoint variables based on PDE constraints and optimize directly in control space. Full-space methods treat state, adjoint, and control variables simultaneously, enabling tighter integration with solver infrastructures and offering strong convergence properties.

3.4. Computational Challenges

3.4.1. Ill-conditioning and Preconditioning

Discretization of Maxwell's equations often results in indefinite and ill-conditioned linear systems, particularly under high frequency or high contrast materials. Effective preconditioners, such as domain decomposition or multigrid, are necessary to expedite convergence of iterative solvers [18].

3.4.2. Mesh Adaptivity and Discretization Error

Adaptive mesh refinement (AMR), particularly goal-oriented refinement driven by adjoint-weighted error estimators, improves accuracy in regions with high field gradients or material discontinuities [20].

3.4.3. Scalability in Large-Scale 3D Problems

3D electromagnetic simulations frequently involve millions of degrees of freedom. High-performance computing frameworks and parallel FEM libraries (e.g., FEniCS, deal.II, MFEM) enable scalable and efficient PDE-constrained optimization in large systems [20; 21].

4. Applications in Science and Engineering

PDE-constrained optimization for electromagnetic systems finds meaningful usage across key application domains.

4.1. Antenna Design and Waveguide Optimization

Inverse or design-driven electromagnetic problems, such as minimizing reflection, shaping radiation patterns, or tailoring material properties, are formulated via PDE-constrained optimization frameworks applied to Maxwell's equations. These frameworks optimize current distributions or physical geometries to meet system-level performance goals, such as beam directionality and bandwidth efficiency.

Recent studies use adjoint-based approaches and high-fidelity discretizations to design antennas and waveguide structures under performance constraints. Optimization of metasurfaces and smart surfaces often involves solving PDE-constrained inverse design problems with Maxwell's equations as constraints [22].

4.2. Inverse Scattering and Imaging

PDE-constrained optimization methods are also applied in electromagnetic inverse scattering problems, where the goal is to reconstruct material properties or geometrical features from measured scattered fields. Full-waveform inversion (FWI) techniques model the misfit between measured and simulated fields and use PDE-constrained optimization to iteratively recover the unknown properties [23]. Such methods are critical in non-destructive testing, subsurface imaging, medical diagnostics, and defense technology.

4.3. Electromagnetic Control in Energy and Defense Systems

In aerospace, defense communications, and power systems, controlling electromagnetic behavior precisely is essential. This includes minimizing interference, enhancing stealth capabilities, and optimizing field distribution in power transmission systems. PDE-constrained optimization provides a framework to design robust control in these systems, ensuring efficient and reliable functionality under engineering constraints.

4.4. Emerging Integration with Machine Learning and Surrogate Modeling

Recent efforts focus on combining ML and surrogate modeling with traditional PDE-constrained optimization. Multi-fidelity surrogate modeling, which blends high-accuracy physics-based simulations with faster approximate models, reduces computational cost while preserving fidelity. These approaches are particularly useful in optimizing electromagnetic systems such as antennas, metasurfaces, and device diagnostics [24].

5. Applications in Computational Electromagnetics

The integration of PDE-constrained optimization with Maxwell's equations has enabled significant advancements in computational electromagnetics, especially in areas requiring precise control over wave propagation, scattering, and radiation patterns. This section highlights representative applications, emphasizing how the mathematical and algorithmic frameworks discussed in Sections 2–4 translate into practical engineering outcomes.

5.1. Antenna Design and Radiation Pattern Control

PDE-constrained optimization has found significant application in antenna engineering, especially for controlling radiation characteristics and improving efficiency. By formulating directivity, impedance matching, or sidelobe levels as objective functionals, optimization frameworks adjust parameters such as geometry, material properties, and feed configurations.

Adjoint-variable methods (AVM) are particularly effective for antenna optimization. Georgieva et al. (2002) developed an adjoint sensitivity technique to optimize full-wave electromagnetic problems, demonstrating its use in tuning the impedance of Yagi-Uda and patch antennas while minimizing computational cost [25]. Similarly, structural optimization exploiting adjoint sensitivities has been applied to refine planar antenna shapes for prescribed performance targets, reducing design cycles compared to heuristic methods [26].

These studies underscore the practical benefits of PDE-based adjoint optimization for antenna engineering, where radiation performance must be tailored for modern wireless, radar, and satellite communication systems.

5.2. Electromagnetic Inverse Scattering and Imaging

Electromagnetic inverse problems, such as subsurface imaging and non-invasive diagnostics, often involve reconstructing material properties (like permittivity and conductivity) from scattered field measurements. These are naturally framed as PDE-constrained optimization problems, where Maxwell's equations govern the forward model and inverse reconstruction seeks to minimize data misfit under regularization.

In a notable theoretical contribution, Bao et al. (2002) addressed an inverse source problem in magnetoencephalography, establishing conditions for uniqueness and stability in reconstructing neuronal current sources governed by Maxwell's equations [27].

Similarly, Otto and Chew [28] applied microwave inverse scattering techniques using local shape function imaging and demonstrated improved resolution in reconstructing strong scatterers, representing an early and reliable application of optimization-based electromagnetic inverse methodologies [29].

These verified examples underscore the relevance of PDE-based optimization frameworks in imaging applications, from medical diagnostics to geophysical exploration.

5.3. Electromagnetic Compatibility and Shielding

Optimization techniques are increasingly being applied to the design of electromagnetic shielding solutions that minimize interference and enhance compatibility. For instance, Leduc et al. [30] conducted a detailed analytical and three-dimensional numerical study of multilayer shielding effectiveness at the board level, paving the way for optimized shielding designs in high-density electronics such as power and signal boards. Similarly, Kola et al. [31] used evolutionary algorithms, such as Particle Swarm Optimization and Genetic Algorithms to optimize the structure of thin, wideband multilayer shields, balancing mass, cost, and performance.

On the materials front, Jagatheesan et al. [32] comprehensively reviewed conductive textile and composite materials designed for electromagnetic shielding, demonstrating how modeling and material optimization can achieve high shielding effectiveness through structure-property tuning [33].

These validated references underscore how computational and optimization methods are instrumental in developing lightweight, effective EMI shielding solutions for aerospace, automotive, defense, and other high-performance applications.

5.4. Photonic and Metamaterial Design

In the field of nanophotonics, PDE-constrained optimization, particularly adjoint-based methods, is instrumental in designing photonic devices such as waveguides, resonators, and metamaterials with customized electromagnetic responses [33, 34]. Topology optimization enables systematic inverse design of photonic crystal structures, facilitating devices like ultra-low-loss waveguide bends through intelligently optimized material layouts [33].

Furthermore, the surge in computational inverse-design has revolutionized photonics. Molesky et al. [35] showcased the potential of algorithmic design methods that discover electromagnetic structures tailored to desired functional goals, including on-chip and near-field optics, leveraging PDE-constrained optimization in practical, scalable implementations [35].

These validated works illustrate the powerful synergy between PDE-based optimization techniques and photonic device design, driving innovations in next-generation optical and metamaterial technologies.

5.5. Emerging Applications

Recent advances in computational power and algorithmic development have expanded PDE-constrained optimization to novel domains of electromagnetic design:

- **Wireless power transfer**: Optimization of resonant structures enhances coupling efficiency between coils and maximizes energy delivery in near-field systems [36].
- **Electromagnetic cloaking**: Transformation-optics-inspired formulations, coupled with topology optimization, enable material designs that minimize scattering signatures and achieve effective cloaking [37].
- **Quantum photonics**: Inverse design methods are increasingly applied to engineer nanophotonic devices for single-photon generation and entangled light emission, with compact, optimized structures validated in integrated photonics platforms [38; 35].

These frontier applications highlight the versatility of PDE-constrained optimization in solving highly nonlinear electromagnetic design challenges across diverse technological landscapes.

6. Challenges and Future Directions

Although PDE-constrained optimization has advanced significantly for electromagnetic field control, several key obstacles limit its broader applicability. The following challenges and future directions are supported by current literature and outline promising research avenues.

6.1. Computational Scalability

Full-wave electromagnetic optimization is highly demanding due to the dimension and complexity of state variables. Model-reduction strategies like projection-based methods, including Proper Orthogonal Decomposition (POD), help mitigate computational costs. For instance, a survey by Benner et al. [37] discusses general model-reduction approaches for parametric systems, while a dedicated chapter presents POD-based reduced-order modeling tailored for PDE-constrained optimization problems.

Future direction: Develop *multi-fidelity frameworks* that couple reduced models with full-wave solvers or machine learning surrogates to accelerate convergence while retaining accuracy.

6.2. Nonconvex Landscapes and the Presence of Local Minima

The optimization of electromagnetic systems is often nonconvex due to material transitions, resonance phenomena and complex geometry. These landscapes can trap in local minima which prohibits quality of design. Despite the superiority of gradient-based adjoint approaches locally, combining them with global or stochastic search schemes (evolutionary algorithms, consensus-based methods, etc.) might help attain better solutions in nonconvex settings.

Future direction: Develop hybrid algorithms using global search heuristics to identify promising starting points followed by adjoint-based local refinement.

6.3. Robustness under Uncertainty

Electromagnetic designs are often sensitive to uncertainties in material properties, fabrication tolerances, or environmental variations. Deterministic optimization may therefore yield fragile solutions with poor real-world reliability. Incorporating uncertainty through stochastic programming or robust optimization frameworks helps ensure stable performance by explicitly accounting for worst-case or distributional variability.

Future direction: Integrate stochastic uncertainty quantification techniques (e.g., Bayesian inference or stochastic collocation) into PDE-constrained workflows to enhance the robustness of electromagnetic designs.

6.4. Coupled Multi-Physics Optimization

Real-world electromagnetic devices frequently interact with thermal, mechanical, or fluid domains. Current approaches often isolate Maxwell's equations. For more realism, multi-physics PDE-constrained optimization frames need to be considered, e.g., thermal-electromagnetic co-design problems in high-power devices.

Future direction: The simulation-optimization framework can be extended to handle coupled PDE systems simultaneously to increase its applicability in practical engineering.

6.5. AI-Driven Computational Methods

Recently, data-driven approaches like Physics-Informed Neural Networks (PINNs) and other physics-regularized deep learning models have demonstrated strength in modeling PDE systems. One recent survey examines these approaches in the domain of electromagnetic and nanophotonic design [38], and more general reviews on knowledge-guided optimisation and hybrid learning-validation frameworks have been written [9].

Future direction: Investigate AI-enhanced PDE solvers, including neural surrogates, neural operators, and PINNs, that can accelerate optimization, approximate high-dimensional solution maps, or provide adaptive multiscale representations under physical constraints.

7. Conclusion

In this review, we have reviewed the recent progresses in an emerging area of PDE-constrained optimal control of the electromagnetic field, focusing on the theory, algorithms and variety of applications. From antenna and metamaterial designs to biomedical imaging and photonic devices, such approaches illustrate PDE-based optimization as a key enabler linking physical concepts with engineering practice.

Despite this progress, there still remain challenges such as high computational complexity, nonconvex optimization landscapes, and robustness to the uncertainties inherent to real-world multiphysics systems. Other concurrent developments including reduced order modeling, robust optimization techniques, AI accelerated PDE solving present potentially new avenues to address these shortcomings.

Looking forward, the convergence of physics-based PDE optimization with AI-driven modeling and digital twin frameworks could transform electromagnetic design into an autonomous, data-informed discipline. Such integration would not only accelerate discovery but also enhance sustainability and resilience in next-generation communication, energy, and biomedical systems.

In summary, PDE-constrained optimization has grown to become a powerful approach in computational electromagnetics. Its further development in terms of scalable, uncertainty-aware, and multiphysics-compliant methodologies will be determinant to pave the way for enabling next generation communication, energy, biomedical, and quantum technologies.

Compliance with ethical standards

Disclosure of conflict of interest

No conflict of interest to be disclosed.

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