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(Review Article)

Impact of Singular Value Decomposition: A review study

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Abstract

Singular Value Decomposition (SVD) is a fundamental matrix factorization technique that provides deep insight into the structure of linear systems. It decomposes a given matrix into orthogonal and diagonal components, enabling the identification of key features such as rank, range, and noise characteristics. This paper discusses both the computational methods for obtaining the SVD and its wide-ranging applications across science and engineering.

Keywords: Singular Value Decomposition; Eigen values; Rank; Transform

1. Introduction

Singular Value Decomposition (SVD) is a fundamental mathematical technique in linear algebra that decomposes a matrix into three component matrices representing its intrinsic geometric structure. In imaging applications, this decomposition separates signal components based on their relative strength or significance, allowing selective suppression of noise and enhancement of meaningful features. By isolating dominant singular values associated with true signal content and filtering out smaller singular values corresponding to noise, SVD provides an effective framework for improving image quality. Its versatility and interpretability have made it a valuable tool in various domains, including signal processing, pattern recognition, and medical image reconstruction[1, 2].

In the context of medical imaging, particularly in angiography, SVD has emerged as a promising method for improving the accuracy of quantitative measurements. The technique helps stabilize image-derived parameters by mitigating the effects of acquisition noise, injection variability, and motion artifacts—common challenges in vascular imaging[3, 4]. Unlike conventional filtering methods that may blur fine vascular details, SVD preserves spatial and temporal integrity while enhancing the signal-to-noise ratio (SNR). As a result, it enables more consistent extraction of hemodynamic and morphological features critical for clinical assessment. Given its data-driven nature and adaptability, SVD offers a robust foundation for developing standardized and reproducible image-processing pipelines in both research and clinical settings[5-7].

This paper takes the SVD formulation from Mondal et al.'s [8] paper on deconvolutional methods using SVD and brings out the analysis on the mathematical formulation of the method. SVD helps to reduce noise and variability, improve the signal-to-noise ratio (SNR), and enhance the reliability of vascular measurements in 2D angiography. The findings of this review help understanding the concept of SVD which can be helpful for both clinical practice and research[9, 10].

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2. Materials and Methods

2.1. Definition

Singular Value Decomposition (SVD) is a matrix factorization technique that expresses any real (or complex) matrix as the product of three special matrices:

$$A = U \sum V^T$$

where:
A is an m x n matrix
U is an m x m orthogonal matrix ($U^TU = I_m$)
V is an n x n orthogonal matrix ($V^TV = I_n$)
$\sum$ is a m x n diagonal matrix whose diagonal entries are non-negative and called singular values.

Mathematically,

$$\Sigma = \begin{bmatrix} \sigma_1 & 0 & 0 & \cdots & 0 \\ 0 & \sigma_2 & 0 & \cdots & 0 \\ 0 & 0 & \sigma_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \sigma_p \end{bmatrix}$$

Where $\sigma_1 \geq \sigma_2 \geq \sigma_3 \ldots \geq \sigma_p \geq 0$, and $p = \min(m,n)$.

2.2. Mathematical derivation

To find the SVD, start with the symmetric matrices A^TA and AA^T :

- Compute eigen values and eigen vectors of A^TA:

$$A^TA\, v_i = \lambda_i\, v_i$$

Where $\lambda_i \geq 0$ are eigen values and v_i are eigen vectors of A^TA

Define $\sigma_i = \sqrt{\lambda i}$ these are the singular values

- Similarly, $A\, A^T\, u_i = \lambda_i\, u_i$ gives singular vectors u_i

Hence,

$$A\, v_i = \sigma_i\, u_i\, ,\; A^T u_i = v_i\, \sigma_i$$

Assembling u_i into u, all v_i into V, and all σ_i into $\sum$, we get:

$$A = U \sum V^T$$

2.3. Geometric Interpretation

SVD can be interpreted geometrically as:

$$A : R^n \quad R^m \rightarrow$$

transforms the unit sphere in R^n into an ellipsoid in R^m.
V rotates (or reorients) the coordinate system in input space.
Σ stretches or compresses along orthogonal directions by factors σ_i
U rotates again to align the output in the target space.

2.4. Rank and Approximation

If the rank of A is r, then only the first r singular values are nonzero.

SVD provides the best low-rank approximation to A in terms of the Frobenius norm or 2-norm:

$$A_r = \sum_{i=1}^{r} \sigma_i u_i v_i^T$$

This A_r minimizes

$$\|A - A_r\|_F$$

Among all rank-r matrices.

This property is the foundation of Principal Component Analysis (PCA) and image compression.

2.5. Numerical stability and Applications

Because SVD works even for non-square and ill-conditioned matrices, it's numerically stable and widely used in:

- Least-squares solutions:

For Ax = b,

$$x = V\Sigma^{-1}U^T b$$

- Noise reduction and filtering: remove components corresponding to small singular values.
- Data compression and PCA: retain only top singular values.
- Image denoising, medical imaging (CT/MRI reconstruction), and signal processing[11].

Example:

For example: if

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

Then

$$A^T A = \begin{bmatrix} 10 & 6 \\ 6 & 10 \end{bmatrix}$$

Whose eigen values are $\lambda_1 = 16$, $\lambda_2 = 4$.

Hence, $\sigma_1 = 4$, $\sigma_2 = 2$.

The SVD of A can be written as:

$$A = U \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} V^T$$

Showing how A stretches space by 4x and 2x along orthogonal directions.

Limitations

Some of the limitations of SVD are as follows:

- Computing the SVD via the eigen value decomposition of A^TA is numerically unstable because it squares the condition number of A [12].
- SVD is computationally expensive, especially for large matrices.
- When dealing with ill-conditioned matrices, small perturbations in data can lead to large changes in the smallest singular values.
- Full SVD requires storage of three large matrices U, Σ, V^T.
- For high-dimensional problems, this results in substantial memory usage.
- Efficient partial or truncated SVD algorithms exist, but their implementation is complex and sensitive to numerical precision.

The accuracy of small singular values is limited by machine arithmetic. For example, a matrix with a smallest singular value on the order of 10^{-12} may appear as 10^{-8} or vanish entirely, depending on floating-point representation, which can mislead interpretation of rank or stability.

While SVD is a powerful and elegant mathematical tool for analyzing matrix structure, its practical limitations stem from computational cost, numerical sensitivity, and finite precision effects on modern computing machines

3. Conclusion

Singular Value Decomposition breaks a matrix into its most fundamental components — rotations and scalings — revealing how it transforms space. Mathematically elegant and computationally stable, SVD underpins modern applications from AI-driven imaging to recommendation systems and genome analysis.

Compliance with ethical standards

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Disclosure of conflict of interest

No conflict of interest to be disclosed.

Statement of ethical approval

The study was approved by the Institutional Ethics Committee

Statement of informed consent

Informed consent was obtained from all participants included in the study

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