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Viscous and Joule Dissipations in Hydro-Magnetic Flow of a Nanofluid with Homogeneous-Heterogeneous Reactions and Heat Absorption

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Abstract

Present paper deals with the study of boundary layer hydromagnetic nano-fluid flow past a stretching sheet under the influence of magnetic field, homogeneous-heterogeneous reactions, heat absorption, viscous dissipation, and Joule dissipation. The non-linear partial differential equations, governing the present flow model, are transformed into a system of non-linear ordinary differential equations (ODE) using the suitable similarity transformation. These transformed ODEs are, then solved using Spectral Relaxation Method (SRM). In order to validate the results obtained by the SRM method, the numerical values of skin-friction co-efficient, rate of heat and mass transfers have been calculated, for different values of magnetic parameter M and Smidth number Sc, using bvp4c routine of MATLAB and found that these values are in excellent agreement with the values calculated through SRM technique. Numerical values of velocity, temperature and concentration distribution for various values of flow parameters have been calculated and presented in graphical forms whereas those of skin-friction co-efficient, Nusselt number at the surface are supplied through the table.

Keywords: Hydro-magnetic; Nano-fluid; Homogeneous-Heterogeneous Reactions; Stretching Sheet; Viscous and Joule Dissipations; Heat Absorption.

1. Introduction

Incompressible, viscous fluids suspended with nano-sized solid particles have been thoroughly studied because they are used in heat transfer devices. Convective heat transfer rates and nanoparticle suspension have an impact on the thermal conductivity of various engineering devices. Buongiorno [1] investigated that the nanofluids can speed up the pace of heat transfer in particular situations, such as in nuclear reactor systems and in a number of heat transfer devices. These can also be utilized as coolants. First, Choi et al. [2] have shown that adding nanoparticles can increase the thermal conductivity of the base fluid by up to two times. Das and Choi [3], Wang and Mazumdar [4], as well as Das et al. [5] have all examined heat transmission in nanofluids. These fluids are used in a variety of engineering processes, including plastic sheet extrusion, continuous stretching of plastic films and synthetic fibers, metallic plate cooling, and polymer extrusion. This makes the study of viscous and incompressible nanofluids over stretching or shrinking sheets incredibly fascinating. Continuous strip or filament cooling is a typical stage in metalworking operations, such as when copper wires are drawn, annealed, and thinned. Ibrahim et al. [6] study is one of the few to look at how an external magnetic field affects how Nanofluid flows and transfers heat to an extended sheet near the stagnation point. Convective heat transfer is used in many engineering systems because it directly affects how the nanofluids flow changes as the heat transmission varies. Kuznetsov and Neild [7] examined the flow of a nanofluid through a vertical plate using natural boundary-layer convection. Analytical methods were successful in resolving heat transfer and flow problems. Using

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mixed convection stagnation point flow, Das [8] examined the transfer of heat and flow of a copper-water Nanofluid approaching a contracting sheet.

The natural convection of nanofluids and Brownian motion are among the reasons why the assumption that temperature and density fluctuate linearly is invalid. This assumption, commonly used in research publications such as Pathra [9], fails to account for significant density variations caused by heat generated through viscous dissipation and thermal stratification (as in wall jets). There is a significant difference in temperature between the fluid surface and the surface of the surrounding plate. The viscous and Joule dissipation effects, as well as the density-temperature relationship nonlinearity all have a major impact on the flow of the boundary layer. Variations in heat transfer within the fluid produced by changes in the velocity gradient are examples of factors that have a direct influence on the heat transfer properties of the flow. Kameswaran et al. [10] studied the flow of a hydromagnetic nanofluid influenced by viscous dissipation and chemical reaction effects through a stretching or contracting sheet. The issue of nanofluid flow was explained using a model for the nanoparticle volume fraction. Along with the unstable viscous fluid flow, the incompressible nanofluid boundary layer and the Magnetohydrodynamic (MHD) effects of convection, viscous dissipation, and thermal radiation along a stretching sheet were also be taken into consideration.

Hady et al. [11] studied the impact of thermal radiation on nanofluid flow over nonlinearly stretching sheets, considering viscous dissipation in heat transfer. Using the concept of Newtonian heating, Olanrewaju and Makinde [12] investigated the flow of a nanofluid across a porous flat surface near the boundary-layer stagnation point. In the context of nanofluids, Brownian motion and thermophoresis processes were discussed. Catalysis, combustion, and biological processes are examples of systems involving both homogeneous and heterogeneous reactions. The homogeneous reaction, which predominates in most fluids, interacts with the heterogeneous reaction that occurs on specific catalytic surfaces. The outcome of this interaction can significantly affect the rates of production and consumption of reactant species both within the liquid and on the catalyst surfaces. Merkin [13] discovered an isothermal homogeneous–heterogeneous reaction mode for a viscous fluid in boundary-layer flow over a flat plate. He used cubic autocatalysis to describe the homogeneous and heterogeneous reactions as the predominant mechanisms near the leading edge of the plate. The homogeneous–heterogeneous reaction in the boundary layer was further examined by Chaudhary and Merkin [14]. Khan and Pop [15] examined the flow impacts on a wall that is infinitely porous and has a homogeneous-heterogeneous composition response at the two-dimensional stagnation point. Khan and Pop [16] looked on how a viscoelastic fluid approaching a stretched sheet is affected by a homogeneous-heterogeneous interaction. As the viscoelastic parameter grew, they saw that the concentration at the surface dropped. Kameswarm et al. [17, 18] researched the homogeneous-heterogeneous responses. Many researchers [19-35] contributed in this direction considering different aspects of the problem.

Objective of the current study is to offer a thorough discussion on the effects of heat absorption, viscous and Joule dissipations on hydro-magnetic nanofluid flow past a stretchable surface taking homogeneous-heterogeneous reactions into account.

2. Formulation of the problem

Consider a two-dimensional boundary layer flow of a viscous, incompressible and electrically conducting nanofluid over a stretching/shrinking surface. A consistent transverse magnetic field B_0 permeates the fluid flow. Since the applied magnetic field is dominant over the induced magnetic field for the fluids with low magnetic Reynolds numbers [36] so the induced magnetic field is neglected here. The polarization of the magnetic field has no impact because no external electric field exists [37]. The temperature balance between the base fluid and the nanoparticles prevents them from slipping together. The geometry of the problem is shown in figure - 1. Table 1 shows the thermo-physical characteristics of the base fluid and nanoparticles.

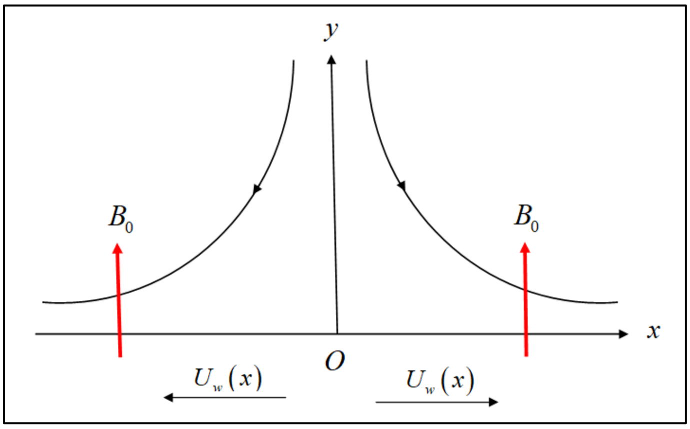


Figure 1 Geometry of the problem

Table 1 Thermo-Physical Characteristics of Water and Nanoparticle

	$\rho(\mathrm{kg/m^3})$	$C_p\left(\frac{J}{\mathrm{kgK}}\right)$	$k(W/\mathrm{mK})$
Pure water	997.1	4179	0.613
Copper (Cu)	8933	385	401
Gold (Au)	19282	129	310

The homogeneous reaction takes place between the species A and B as

$$A + 2B \rightarrow 3B, \quad \text{rate} = k_c a b^2 \tag{1}$$

while a heterogeneous first order, isothermal reaction undergoes on the catalyst surface, as

$$A \rightarrow B, \quad \text{rate} = h_s a \tag{2}$$

where k_c and h_s are homogeneous and heterogeneous reaction rates, respectively [14, 38].

The flow and heat transfer of nano-fluid are approximated by the following equation, that uses the Cartesian coordinates x and y to express mass, momentum, energy, and concentration:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho_{nf}}u \tag{4}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} + \frac{\sigma {B_0}^2}{(\rho c_p)_{nf}}u^2 + \frac{\mu_{nf}}{(\rho c_p)_{nf}}\left(\frac{\partial u}{\partial y}\right)^2 - \frac{Q_0}{(\rho c_p)_{nf}}(T - T_\infty) \tag{5}$$

$$u\frac{\partial a}{\partial x} + v\frac{\partial a}{\partial y} = D_A\frac{\partial^2 a}{\partial y^2} - k_c a b^2 \tag{6}$$

$$u\frac{\partial b}{\partial x} + v\frac{\partial b}{\partial y} = D_B\frac{\partial^2 b}{\partial y^2} + k_c a b^2 \tag{7}$$

where T is the temperature of the nanofluid. Velocity components along x and y directions are represented by u and v respectively. μ_{nf}, σ, ρ_{nf}, α_{nf}, Q_0 are, respectively, viscosity, electrical conductivity, density, thermal diffusivity, and heat absorption.

The dynamic viscosity μ_{nf}, effective density ρ_{nf}, thermal diffusivity α_{nf}, thermal conductivity k_{nf}, and heat capacitance $(\rho c_p)_{nf}$ of the nanofluid are defined as [17]:

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}} \tag{8}$$

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s. \tag{9}$$

$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}} \tag{10}$$

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + 2\phi(k_f - k_s)} \tag{11}$$

$$(\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_s \tag{12}$$

The subscripts f and s, denote the physical properties of the base fluid and nanoparticles. ϕ denotes the volume fraction of the nanoparticles

The boundary conditions of the problem are

$$u = U_w(x) = cx, v = 0, T = T_w, D_A\frac{\partial a}{\partial y} = h_s a, D_B\frac{\partial b}{\partial y} = -h_s a \text{ at } y = 0 \tag{13}$$
$$u \to 0, T \to T_\infty, a \to a_0, b \to 0 \text{ as } y \to \infty \tag{14}$$

The following transformation is now being introduced:

$$\psi(x,y) = \sqrt{c\nu_f}\,xf(\eta), \theta = \frac{(T-T_\infty)}{(T_w-T_\infty)}, a = a_0 g(\eta), b = a_0 h(\eta) \text{ where } \eta = \sqrt{\frac{c}{\nu_f}}\,y \tag{15}$$

The transformation indicated above as well as the dimensionless stream function η are selected such that

$u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$.

Using the above transformation (15), the equation (3) is satisfied automatically and equations (4) - (7) assumes the form, we get because the transformation mentioned above satisfies the equation of continuity (3).

$$\frac{1}{\phi_1}f''' + ff'' - f'^2 - \frac{M}{\phi_2}f' = 0 \tag{16}$$
$$\frac{1}{Pr}\frac{\phi_4}{\phi_3}\theta'' + f\theta' + \frac{M}{\phi_3}Ecf'^2 + \frac{Ec}{\phi_3(1-\phi)^{2.5}}f''^2 - \frac{\beta_h}{\phi_3}\theta = 0 \tag{17}$$
$$\frac{1}{Sc}g'' + fg' - Kgh^2 = 0 \tag{18}$$
$$\frac{\delta}{Sc}h'' + fh' + Kgh^2 = 0 \tag{19}$$

where

$$\beta_h = \frac{Q_0}{c(\rho c_p)_f}, \mathrm{Pr} = \frac{\nu_f}{\alpha_f}, \mathrm{Ec} = \frac{U_w^2}{\Delta T(c_p)_f}, \mathrm{Sc} = \frac{\nu_f}{D_A}, \mathrm{M} = \frac{\sigma {B_0}^2}{c\rho_f}, \mathrm{K} = \frac{k_c a_0^2}{c}, \quad \phi_1 = (1-\phi)^{2.5}\left\{(1-\phi) + \phi\left(\frac{\rho_s}{\rho_f}\right)\right\},$$

$$\phi_2 = \phi_1(1-\phi)^{-2.5}, \ \ \phi_3 = \left\{(1-\phi) + \phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right\}, \phi_4 = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + 2\phi(k_f - k_s)}, \ \text{ and } \delta = \frac{D_B}{D_A}$$

The non-dimensional parameters defined above are, Prandtl number Pr, Eckert number Ec, magnetic parameter M, Schmidt number Sc, heat absorption parameter β_h , homogeneous reaction rate K, and the ratio of mass diffusion constants δ. The functions ϕ_1, ϕ_2, ϕ_3 and ϕ_4 depend on the thermal properties of the nanoparticles and the base fluid, and are dimensionless.

According to the transformation (15), the boundary conditions (13) and (14) have been reduced as

$$f(\eta) = 0, f'(\eta) = 1, \theta(\eta) = 1, g'(\eta) = K_s g(\eta) \text{ at } \eta = 0; \quad (20)$$

$$f'(\eta) \to 0, \theta(\eta) \to 0, g(\eta) \to 1, h(\eta) \to 0 \text{ as } \eta \to \infty, \quad (21)$$

where $K_s = \frac{h_s}{D_A}\sqrt{\frac{\nu_f}{c}}$ is the strength of the heterogeneous reaction.

We also assume that the diffusivity of chemical species A and B are equal i.e. $\delta = 1$.

The following relationship results from this supposition:

$$g(\eta) + h(\eta) = 1. \quad (22)$$

Under this assumption, Eqs. (18) and (19) are reduced to

$$\frac{1}{Sc} g'' + f g' - k g (1-g)^2 = 0, \quad (23)$$

as well as constrained by boundary conditions

$$g'(0) = K_s g(0), g(\eta) \to 1 \text{ as } \eta \to \infty. \quad (24)$$

Other physical quantities of physical interest are the local skin friction C_f and the local Nusselt Number Nu_x are as below:

$$C_f Re_x^{\frac{1}{2}} = \frac{1}{(1-\phi)^{2.5}} f''(0), \quad (25)$$

$$Nu_x / Re_x^{1/2} = -\phi_4 \theta'(0) \quad (26)$$

3. Numerical procedure and validation

Equations (16), (17) and (23) are solved together with the boundary conditions (20), (21) and (24) using the Spectral Relaxation Method (SRM) suggested by Kameswaran et al. [39]. The iteration scheme is obtained in the framework of SRM as

$$f'_{r+1} = p_r, f_{r+1}(0) = 0 \quad (27)$$

$$\frac{1}{\phi_1} p''_{r+1} + f_{r+1} p'_{r+1} - \frac{M}{\phi_2} p_{r+1} = p_r^2 \quad (28)$$

$$\frac{\phi_4}{Pr\phi_3} \theta''_{r+1} + f_{r+1}\theta'_{r+1} - \frac{\beta_h}{\phi_3}\theta_{r+1} = -\frac{MEc}{\phi_3} f'^{2}_{r+1} - \frac{Ec}{\phi_3(1-\phi)^{2.5}} f''^{2}_{r+1} \quad (29)$$

$$\frac{1}{Sc} g''_{r+1} + f_{r+1} g'_{r+1} - K g_{r+1} = K(g_r^3 - 2g_r^2) \quad (30)$$

The boundary requirements of iteration scheme are as follows:

$$p_{r+1}(0) = 1, \qquad p_{r+1}(\infty) \to 0 \quad (31)$$

$$\theta_{r+1}(0) = 1, \qquad \theta_{r+1}(\infty) \to 0 \quad (32)$$

$$g'_{r+1}(0) = K_s g_{r+1}(0), \qquad g_{r+1}(\infty) \to 1 \quad (33)$$

We apply the Chebyshev spectral collocation method to solve the decoupled system (27) - (33). Before applying the spectral method, it is convenient to transform the domain on which the governing equation is defined to the interval [-1, 1] on which the spectral method can be implemented. We use the transformation $\eta = L(\xi + 1)/2$ to map the interval [0, L] to [-1, 1], where L is chosen to be large enough to numerically approximate the conditions at infinity. The basic idea behind the spectral collocation method is the introduction of a differentiation matrix D which is used to approximate the derivatives of the unknown variables at the collocation points as the matrix vector product of the form

$\frac{df_{r+1}}{d\eta} = \sum_{k=0}^{N} \mathbf{D}_{lk} f_r(\xi_k) = \mathbf{D}\mathbf{f}_r, l = 0,1,2,\dots,N$ (34) where N + 1 is the number of collocation points (grid points), D = 2D/L, and f = [f(τ_0), f(τ_1), . . . , f (τ_N)]^T is the vector function at the collocation points.

Higher-order derivatives are obtained as powers of D, that is,

$f_r^{(p)} = \mathbf{D}_p \mathbf{f}_r$ (35)

where p represents the order of the derivative.

Using the spectral method to solve equations (27) - (30), we obtain

$$\mathbf{A}_1 \mathbf{f}_{r+1} = \mathbf{B}_1, \quad f_{r+1}(\xi_N) = 0 \tag{36}$$
$$\mathbf{A}_2 \mathbf{p}_{r+1} = \mathbf{B}_2, \quad p_{r+1}(\xi_N) = 1, \quad p_{r+1}(\xi_0) = 0 \tag{37}$$
$$\mathbf{A}_3 \Theta_{r+1} = \mathbf{B}_3, \quad \theta_{r+1}(\xi_N) = 1, \quad \theta_{r+1}(\xi_0) = 0 \tag{38}$$
$$\mathbf{A}_4 G_{r+1} = \mathbf{B}_4, \quad g'_{r+1}(\xi_N) = K_s g_{r+1}(\xi_N), \quad g_{r+1}(\xi_0) = 1 \tag{39}$$

where,

$$\mathbf{A}_1 = \mathbf{D},\ \mathbf{B}_1 = p_r \tag{40}$$
$$\mathbf{A}_2 = \text{diag}\left(\frac{1}{\phi_1}\right)\mathbf{D}^2 + \text{diag}\,(\mathbf{f}_r)\mathbf{D} + \text{diag}\left(-\frac{M}{\phi_2}\right)\mathbf{I}, \mathbf{B}_2 = p_r^2 \tag{41}$$
$$\mathbf{A}_3 = \text{diag}\left(\frac{\phi_4}{Pr\phi_3}\right)\mathbf{D}^2 + \text{diag}\,(\mathbf{f}_r)\mathbf{D} + \text{diag}\left(-\frac{\beta_h}{\phi_3}\right)\mathbf{I}, \mathbf{B}_3 = -\frac{MEc}{\phi_3} f'^2_{r+1} - \frac{Ec}{\phi_3(1-\phi)^{2.5}} f''^2_{r+1} \tag{42}$$
$$\mathbf{A}_4 = \text{diag}\left(\frac{1}{Sc}\right)\mathbf{D}^2 + \text{diag}\,(\mathbf{f}_r)\mathbf{D} + \text{diag}\,(-K)\mathbf{I}, \mathbf{B}_4 = K(g_r^3 - 2g_r^2) \tag{43}$$

In equations (40) - (43), I is an identity matrix.

The initial guesses for the SRM method are as follows:

$$f_0(\eta) = 1 - e^{-y}, \quad p_0(\eta) = e^{-\eta}, \quad \theta_0(\eta) = e^{-\eta}, \quad g_0(\eta) = 1 - 0.5e^{-Ks\eta} \tag{44}$$

In order to validate the results obtained by the SRM method, the numerical values of $-f''(0)$, $\theta'(0)$ and $g'(0)$ have been calculated, for different values of M and Sc, using bvp4c routine of MATLAB and found that these values are in excellent agreement with the values calculated through SRM technique (See Tables 2 and 3). This validates our results are accurate.

Table 2 Findings for Different Magnetic Parameter M Values Compared between SRM and bvp4c When $\boldsymbol{\phi = 0.1, K = 0.5, Ks = 0.5, Ec = 1, h = 2, Sc = 1}$ **and** $\mathbf{Pr = 6.7850}$ **are the Values.**

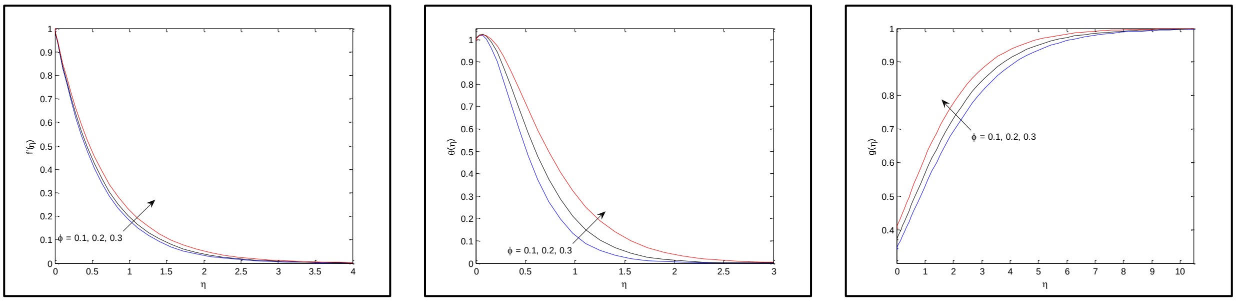
M		**SRM results**		**bvp4c results**		
	−f″(0)	θ′(0)	g′(0)	−f″(0)	θ′(0)	g′(0)
0	1.17474602	-1.80052872	0.23266357	1.17474602	-1.80052872	0.23266357
1	1.46576318	-0.41211132	0.20465203	1.46576318	-0.41211132	0.20465203
2	1.70789202	0.78581494	0.17161545	1.70789202	0.78581494	0.17161545
3	1.91972098	1.85943518	0.11706945	1.91972098	1.85943518	0.11706945
4	2.11039383	2.84292539	0.00000005	2.11039383	2.84292539	0.00000004
5	2.28521237	3.7568641	0.00000001	2.28521237	3.7568641	0.00000001

Table 3 Results of SRM and bvp4c Comparison for Different Schmidt Number Values where $\boldsymbol{\phi = 0.1, Ec = 1, Ks = 0, K = 0, h = 2, M = 2}$ **and** $\boldsymbol{Pr = 6.7850}$ **are the Values.**

Sc		SRM results		bvp4c results		
	$-f''(0)$	$\theta'(0)$	$g'(0)$	$-f''(0)$	$\theta'(0)$	$g'(0)$
1	1.70789202	0.78581494	0.17161545	1.17474602	0.78581494	0.17161545
3	1.70789202	0.78581494	0.31840558	1.70789202	0.78581494	0.31840558
5	1.70789202	0.78581494	0.35997121	1.70789202	0.78581494	0.35997121
7	1.70789202	0.78581494	0.38184183	1.70789202	0.78581494	0.38184183

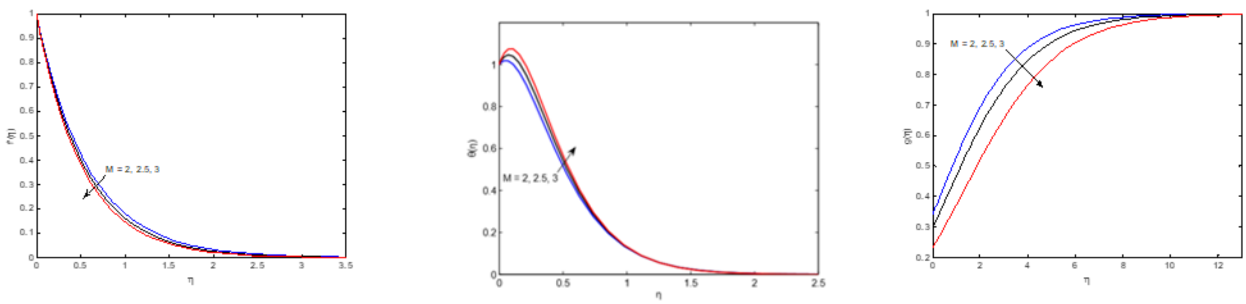
4. Results and discussion

In order to analyze the effects of various flow parameters into the flow pattern, the profiles of nanofluid velocity f', temperatures θ and species concentration g have been depicted for various values of nanoparticle volume fraction parameter ϕ and magnetic parameter M in figures 2 (a) – 3 (c). It is evident from figures 2 (a) - (c) that on increasing ϕ there is an increment in f', θ, and g which represents that the nanoparticle volume fraction have tendency to enhance the nanofluid velocity, nanofluid temperature, and species concentration within the boundary layer region. It is revealed from the figures 3 (a) - (c) that with the increase in M, there is an increment in nanofluid temperature and a decrement is observed in the nanofluid velocity profiles as well as concentration profiles. This implies that the magnetic field have the tendency to induce the nanofluid temperature while it has the reverse effect on the nanofluid velocity as well as species concentration.



a b c

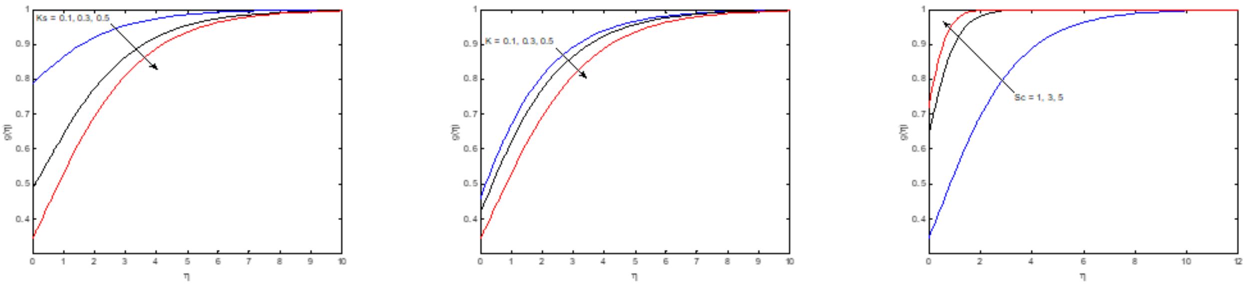
Figure 2 The effect of the solid volume fraction of nanoparticle ϕ on (a) the fluid velocity f', (b) the fluid temperature θ and (c) species concentration g when $M = 2$, $K_s = 0.5$, $K = 0.5$, $Ec = 1$, $\beta_h = 2$, $Sc = 1$ and $Pr = 6.7850$.



a b c

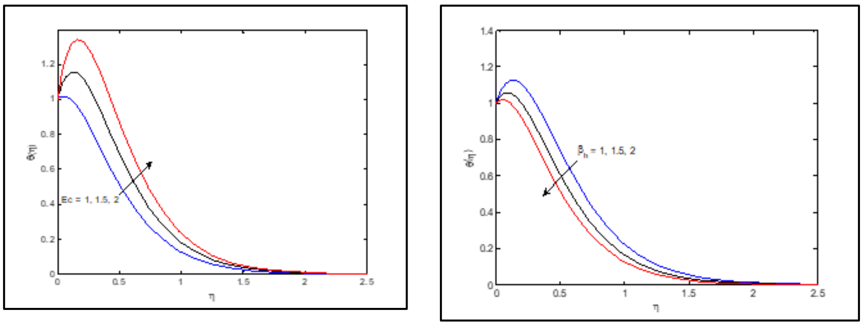
Figure 3 The effect of magnetic parameter M on (a) the fluid velocity f', (b) the fluid temperature θ and (c) species concentration g when $\phi = 0.1$, $K_s = 0.5$, $K = 0.5$, $Ec = 1$, $\beta_h = 2$, $Sc = 1$ and $Pr = 6.7850$.

Figures 4 (a) - (c) represents the impact of heterogeneous reaction parameter K_s, homogeneous reaction parameter K and Schemidt number Sc on species concentration. It can be concluded from the figures 4 (a) - (c) that both homogeneous and heterogeneous reactions act as reducing agent for species concentration while Smidth number behaves like inducing agent for that. The effects of viscous drag and heat absorption on the thermal energy of the nanofluid have been presented in figures 5 (a) - (b). It is observed from figures 5 (a) - (b) that the viscous drag has the ability to increase thermal energy of the nanofluid while heat absorption shows the reverse effect on it.



a b c

Figure 4 The effect of heterogeneous reaction parameter K_s, homogeneous reaction parameter K and Schemidt number Sc on the species concentration g are (a), (b) and (c) respectively, when ϕ = 0.1, M = 2, β_h = 2 and Pr = 6.7850.



a b

Figure 5 The effect of Eckert number Ec and heat absorption parameter β_h on the fluid temperature θ are (a) and (b) respectively, when ϕ = 0.1, M = 2, K_s = 0.5, K = 0.5, Sc = 1 and Pr = 6.7850.

To get an overview of the impact of different physical parameters on the coefficient of skin-friction and rate of heat transfer, numerical values of coefficient of skin-friction and Nusselt number have been calculated and listed in the table below (Table 4). Table 4 reflects that there is increment on $C_f\sqrt{\text{Re}_x}$ and $\frac{Nu_x}{\sqrt{\text{Re}_x}}$ on increasing ϕ and M. This shows that the nanoparticle volume fraction and magnetic field contribute towards the growth of the skin-friction and rate of heat transfer. It is also seen from the table that $\frac{Nu_x}{\sqrt{\text{Re}_x}}$ increases on increasing Ec while decreases on increasing β_h. This reflects that viscous drag induces rate of heat transfer while heat absorption reduces rate of heat transfer.

Table 4 Effect of various parameters on coefficient of skin-friction and Nusselt number when K_s = 0.5, K = 0.5 and Sc = 1.

ϕ	**M**	**β_h**	**Ec**	$C_f\sqrt{\text{Re}_x}$	$\frac{Nu_x}{\sqrt{\text{Re}_x}}$
0.1	2	2	1	2.22256329	1.17647390
0.2	2	2	1	2.83223196	1.76385484
0.3	2	2	1	3.62517456	2.40547217
0.1	**2**	2	1	2.22256329	1.17647390
0.1	**2.5**	2	1	2.36441578	1.99955836

0.1	**3**	2	1	2.49822665	2.78383222
0.1	2	**1.0**	1	2.22256329	3.25896294
0.1	2	**1.5**	1	2.22256329	2.11419889
0.1	2	**2.0**	1	2.22256329	1.1764739
0.1	2	2	**1.0**	2.22256329	1.17647390
0.1	2	2	**1.5**	2.22256329	4.24192195
0.1	2	2	**2.0**	2.22256329	7.30737000

5. Conclusions

A detailed study on boundary layer hydro-magnetic nano-fluid flow past a stretching sheet under the influence of magnetic field, homogeneous-heterogeneous reactions, heat absorption, viscous and Joule dissipations is carried out. Important findings are listed below:

“Nanoparticle volume fraction have tendency to enhance the nanofluid velocity, nanofluid temperature, and species concentration”.

“Both homogeneous and heterogeneous reactions act as reducing agent for species concentration while Smidth number behaves like inducing agent for that”.

“Viscous drag induces rate of heat transfer while heat absorption reduces rate of heat transfer”.

Compliance with ethical standards

Disclosure of conflict of interest

There is no conflict of interest among the authors.

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