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Analytical modelling of diffusion capacitance under monochromatic illumination: Static case

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Abstract

This study proposes an analytical modelling of the diffusion capacitance of a vertical-junction solar cell under monochromatic illumination in the static regime. The developed model incorporates the effects of illumination wavelength, base depth, and applied voltage, enabling an accurate description of the minority carrier dynamics within the junction. The results show that the diffusion capacitance decreases with increasing photon penetration depth and exhibits a nonlinear dependence on voltage. This analytical approach provides an efficient tool for predicting capacitive performance and may guide the optimization of solar cells in specialized photovoltaic applications.

Keywords: Vertical-junction solar cell; Diffusion capacitance; Monochromatic illumination; Analytical modelling; Static regime

1. Introduction

A precise understanding of the capacitive characteristics of solar cells is essential for the optimization of modern photovoltaic devices [1][2]

Among these characteristics, the diffusion capacitance plays a key role in the dynamics of minority carriers and directly influences the efficiency and stability of the cell [3][4][7].

Most previous studies have focused on the experimental or numerical evaluation of capacitance under polychromatic or dynamic regimes [5][6][10][11], leaving a lack of analytical tools to describe the behavior under monochromatic illumination in static conditions. Analytical models have the advantage of providing a direct physical description, allowing a clear identification of the influence of structural and operational parameters on capacitance [8][13][15].

This study therefore proposes an analytical model of the diffusion capacitance of a vertical-junction solar cell operating in the static regime under monochromatic illumination. The main objective is to determine the influence of illumination wavelength, base depth, and applied voltage on capacitance, in order to provide a predictive tool for the design and optimization of solar cells in specialized photovoltaic applications [5][12][13].

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2. Theoretical Model

2.1. Solar Cell Description

The structure under study is a vertical-junction silicon solar cell connected in series (Figure 1). Each elementary cell consists of an n^+ emitter, a space-charge region (SCR), a p-type base, and a heavily doped p^+ region serving as the back collector (Figure 2) [5][13].

The illumination is monochromatic and incident perpendicularly to the front surface (emitter side), along the Z-axis. The photon flux penetration depth in the base is denoted as $\alpha^{-1}(\lambda)$, depending on the wavelength λ [3][14].

The modelling is carried out along the X-axis, perpendicular to the surface of the cell, with:

- x = 0 : emitter /base interface (front),
- x = H: base/collector interface (back).

2.2. Adopted Physical Assumptions

The regime is static (no time dependence).

The illumination is monochromatic with an optical generation described by:

$G(z) = G_o e^{-\alpha z}$ [4]

where G_o is the generation rate at the surface and α the absorption coefficient [4].

Bulk recombination in the base follows a lifetime law with a mean lifetime τ [6][7].

Surface recombination is taken into account both at the front interface (Sf) and the rear interface (Sb).

The Shockley law is applied at the junction (SCR) [7].

2.3. Minority Carrier Diffusion Equation

In the p-type base, the minority carriers are electrons. Their excess density satisfies the diffusion differential equation:

$$D_n \frac{d^2\delta_n}{dx^2} - \frac{\delta_n(x)}{\tau_n} + G(x) = 0$$

Where :

- D_n: electron diffusion coefficient,
- τ_n: minority carrier lifetime,
- $G(x)$: optical generation function [3][11].

2.4. Analytical Solution of Carrier Density

The general solution of this equation is:

$$\delta_n(x) = A\, sinh\left(\frac{x}{L_n}\right) + B\, cosh\left(\frac{x}{L_n}\right) + \frac{G_o e^{-\alpha x}}{\frac{1}{L_n} - \alpha^2}$$

with $L_n = \sqrt{D_n \tau_n}$, the electron diffusion length.

The constants A and B are determined from the boundary conditions.

2.5. Boundary Conditions

At the front interface (x = 0):

$$D_n \frac{d\delta_n(0)}{dx} = S_f \delta_n(0)$$

At the back interface (x = H):

$$D_n \frac{d\delta_n(H)}{dx} = -S_b \delta_n(H)$$

These conditions represent the effect of surface recombination velocities [8][6][11].

2.6. Diffusion Capacitance

The diffusion capacitance is defined as:

$$C_d = q \frac{dQ}{dV}$$

where Q is the stored charge in the base:

$$Q = \int_0^H \delta_n(x) dx$$

The dependence of C_d on applied voltage V, photon penetration depth 1/α(λ), and base thickness HH, is obtained by substituting the analytical solution [5][13] of $\delta_n(x)$ into the integral.

The diffusion capacitance is thus the derivative with respect to V of the stored injected charge, under constant optical parameters.

Limiting cases (rear surface passivated, highly recombinative, thin or thick base) have already been studied in several works [6][7][16][12].

By replacing $\frac{d\delta n_0}{dV}$:

$$C_d(H,V) = \frac{q^2}{KT} \frac{n_i^2}{N_A} e^{\frac{qV}{KT}} \times L_n \frac{\frac{D_n}{L_n} \, sinh\left(\frac{H}{L_n}\right) + S_b \left(cosh\, cosh\, \left(\frac{H}{L_n}\right) - 1\right)}{\frac{D_n}{L_n} \, cos\left(\frac{H}{L_n}\right) + S_b \, sinh\left(\frac{H}{L_n}\right)}$$

$\frac{q^2}{KT} \frac{n_i^2}{N_A} e^{\frac{qV}{KT}}$ and $L_n \frac{\frac{D_n}{L_n} sinh\left(\frac{H}{L_n}\right) + S_b\left(cos\left(\frac{H}{L_n}\right) - 1\right)}{\frac{D_n}{L_n} \, cos\left(\frac{H}{L_n}\right) + S_b \, sinh\left(\frac{H}{L_n}\right)}$

represents the injection component, while the fraction involving hyperbolic functions accounts for geometric and recombination effects.

Minority carrier concentration at the edge of the SCR (Shockley law, low injection):

$\delta_{no}(V) = \frac{n_i^2}{N_A}\left(e^{\frac{qV}{KT}} - 1\right)$, $\frac{d\delta_{no}}{dV} = \frac{q}{KT} \frac{n_i^2}{N_A} e^{\frac{qV}{KT}}$

For simplification, let: $C \equiv cosh\, cosh\, \left(\frac{H}{L_n}\right)$, $L_n = \sqrt{D_n \tau_n}$ et $S \equiv \, sinh\left(\frac{H}{L_n}\right)$

The diffusion capacitance then becomes:

$$C_d(H,V) = \, q \frac{d\delta_{no}}{dV} L_n \frac{\frac{D_n}{L_n} \, S + S_b(C-1)}{\frac{D_n}{L_n} \, + S_b \, S}$$

where q is the elementary charge, D_n the diffusion coefficient, τ_n the lifetime, and $S_b$the back surface recombination velocity.

Useful limiting cases :

Perfectly passivated back surface (S_b→0):

Cd→$\frac{q^2}{KT}\frac{n_i^2}{N_A}L_n e^{\frac{qV}{KT}}\tanh\left(\frac{H}{L_n}\right)$)

Highly recombinative back surface (S_b→∞):

Cd→ $\frac{q^2}{KT}\frac{n_i^2}{N_A}L_n e^{\frac{qV}{KT}}\frac{cosh\frac{H}{L_n}-1}{sinh\frac{H}{L_n}} = \frac{q^2}{KT}\frac{n_i^2}{N_A}L_n e^{\frac{qV}{KT}}\left(coth\frac{H}{L_n} - cosh\frac{H}{L_n}\right)$)

Thick base (H≫Ln): $C_d \approx \frac{q^2}{KT}\frac{n_i^2}{N_A}L_n e^{\frac{qV}{KT}}$

Thin base (H≪Ln): $C_d \approx \frac{q^2}{KT}\frac{n_i^2}{N_A}L_n e^{\frac{qV}{KT}}\frac{H}{1+\frac{S_b H}{2\,D_n}}$

3. Results and Discussion

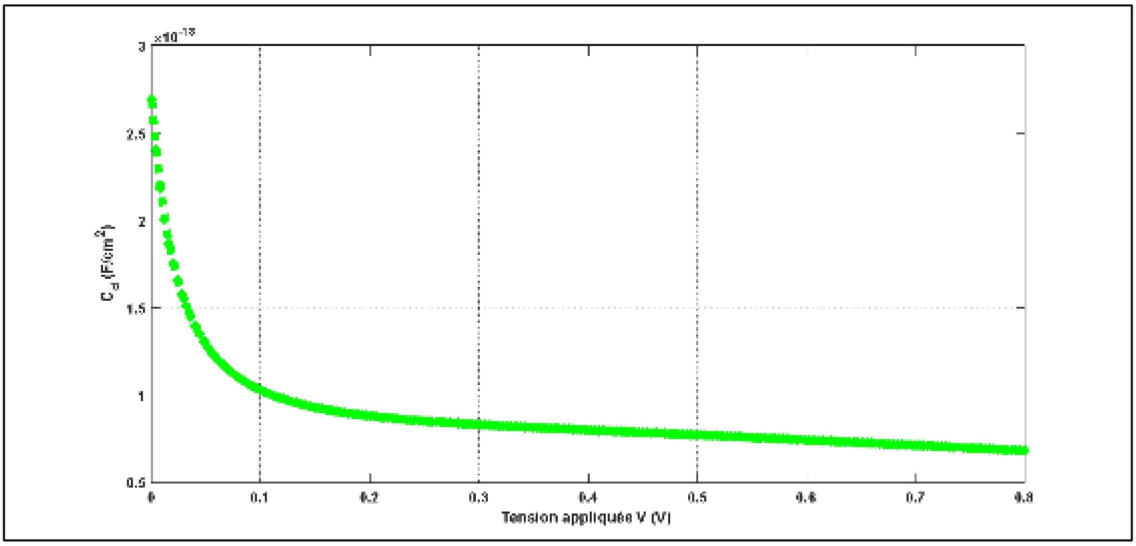

Figure 3 Diffusion capacitance as a function of the applied voltage

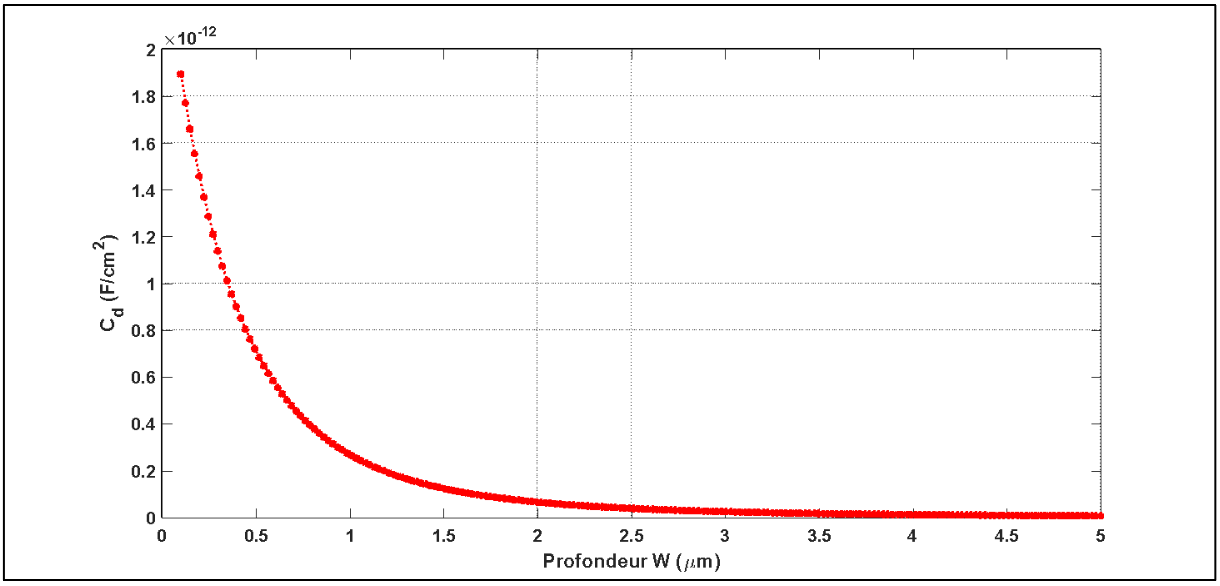

Figure 4 Diffusion capacitance as a function of base thickness

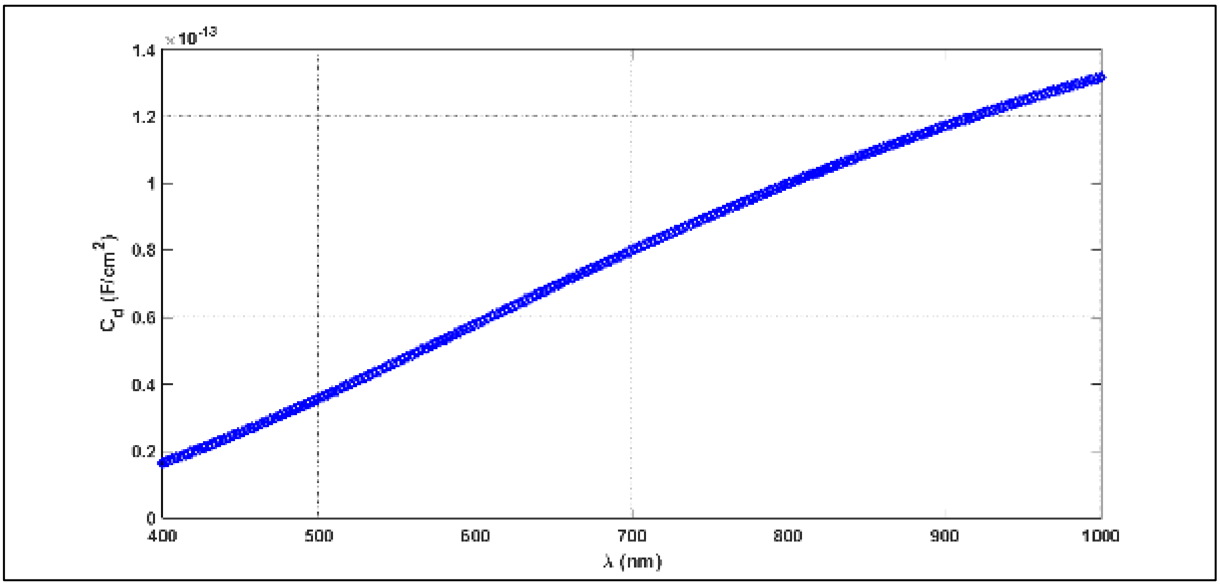

Figure 5 Diffusion capacitance as a function of wavelength

Figure 3 illustrates the evolution of diffusion capacitance as a function of the applied voltage [3][5][7]. A pronounced decrease in Cd is observed when the forward bias increases. This behavior results from the factor $\left(1+\frac{V}{V_T}\right)^{-1}$ included in the analytical expression, which leads to a significant reduction by an order of magnitude as soon as $V \gg V_T$ ($V_T \approx 25.9$ mV at 300 K). Thus, at V=0.8 V, the capacitance drops to about 3% of its initial value under low bias. This decrease reflects, within the adopted model, a reduced capacitive contribution of minority carriers as the forward bias increases. However, it should be noted that in some classical models, diffusion capacitance may instead increase in forward regime, highlighting the need to specify the physical conditions and assumptions retained.

Figure 4 presents the variation of Cd as a function of base thickness [10][13]. The dependence exhibits a non-monotonic shape: capacitance initially increases with W, reaches a maximum for $W \approx 1/\alpha$, and then decreases exponentially beyond this characteristic depth. This behavior can be explained by the existence of an optimal thickness where carrier generation is efficiently coupled to the junction, before deeper absorption and recombination losses lead to a reduction in capacitance. In the considered case (λ=600 nm), this maximum is expected around 0.6 μm.

Figure 5 shows the spectral dependence of diffusion capacitance for a fixed base thickness. The results indicate an increase in Cd with wavelength in the studied range (400–1000 nm) [10][14]. This trend arises from the simplified law $\alpha \sim 1/\lambda$, which favors longer wavelengths where optical penetration is deeper and the capacitive contribution becomes more significant. However, it is important to note that the actual absorption spectrum of silicon differs from this approximation; using tabulated data for $\alpha(\lambda)$ would refine these results and enrich the discussion.

Overall, these three results [5][12][13] highlight the importance of electrical parameters (applied voltage), geometrical parameters (base thickness), and optical parameters (illumination wavelength) in the capacitive behavior of the solar cell. A comparison with experimental data, as well as the introduction of a more realistic absorption coefficient law, would constitute essential extensions to strengthen the validity and applicability of the model.

4. Conclusion

This study analysed the diffusion capacitance of a vertical-junction solar cell in the static regime, highlighting its evolution as a function of the applied voltage, the base thickness, and the wavelength of monochromatic illumination. The results show that the capacitance decreases strongly with forward bias, reaches a maximum for a characteristic depth related to the absorption coefficient, and then increases with wavelength within the studied spectral range.

The proposed model, although simplified, sheds light on the combined influence of electrical, geometrical, and optical parameters on the capacitive behavior of the cell. It thus provides a relevant tool for understanding the internal mechanisms of charge storage and for optimizing the design of photovoltaic devices.

Looking ahead, the developed approach could be extended to the study of polychromatic illumination or dynamic regimes, allowing the exploration of frequency response and transient effects. Such investigations would offer promising perspectives both for improving conversion efficiency and for adapting solar cells to varied illumination conditions.

Compliance with ethical standards

The authors declare that all applicable ethical standards have been followed during the conduct of this research.

Disclosure of conflict of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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