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Compromised payoffs in the presence of incomplete information for supply chain applications

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Abstract

People in situation of conflict are often found to engage based on their own understanding of expected utility from the said conflict. Should people behave completely rational, they would cease to engage the moment there is visibility on one's earnings from the conflict. However, in reality the observable patterns and trends are different given that not always are people engaging rationally. What makes this complex is the fact that many a times a person does not have the required information to be able to engage in a fair play. Distortion in communication and presence of irrationality contribute to the uncertainty present in a game. These are abundantly observed in multi agent interactions such as management of supply chains where information forms a crucial link for the success of the supply chain. This paper proposes a model for non-cooperative interaction between agents who are operating with incomplete information about their opponents. The proposed game models the belief system of the players regarding their opponent as a Markov chain and in doing so incorporates uncertainty in the form of entropy of information. The proposed model is illustrated using a single manufacturer, single supplier game. The results show that in the presence of uncertainty, players are willing to tradeoff a part of their winnings to accumulate as much information about the opponent as they seem satisfactory. This tradeoff is characterized by the fact that the players would have been worse off in the absence of this accumulated information.

Keywords: Incomplete Information; Alternate Games; Rationality; Markov Chain; Manufacturer; Supplier

1. Introduction

Rationality and information characterize a game. With the structural assumptions of rationality and complete information, game theorists focused on obtaining a solution formed by various combinations of strategies available to the players for various types of games. The main challenge in this process arises from the fact that different players have different preferences and thus prefer different outcomes. Conflict of interest starts at this point as each player is looking to answer the question: "What decision would be the best for me?". A player's decision to maximize profit for oneself is dependent on what his opponent's decision will be, which is where the uncertainty in the structure of games is present. Assuming all players play rationally, theorists developed various solution concepts to answers the above question [1]. Such conflicting scenarios exist not only in traditional games (chess, poker, bridge) but also in business, economics, politics, military or even biological situations.

Games have been an essential tool with respect to conflicting multiple agents. One can easily draw a parallel to supply chains owing to the interactive optimization problems encountered in various stages of a supply chain (SC) involving all the agents at every stage of a SC. Even though adoption of games in the domain of operations management was slow initially, the first decade of the 21st century witnessed an academic flare-up in SC management applications of games [2].

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Conventionally, SC management focused on problems concerning the manufacturer and supplier [3, 4], but gradually their focus towards optimization of products and services from the very end of the prime suppliers to customers at the other end where every section, broadly classified into supplier, manufacturer, distributor and customer, have their own facilities. Complexities only increase with increasing number of facilities that need to be managed [5]. Flow of information between the above mentioned four sections became equally important and crucial to business survival. For coordinated movement of materials down the supply chain, an effective communication has to be passed up the chain prior to it making SC management a necessary component of every firm to enhance their competitive advantage [3, 6]. Compromised performances of agents in the supply chain will affect other agents in the interconnected network subject to the severity of the link between these agents [7]. It is appreciable that uncertainty in the chain further increases with such increased complexities. In such situations, structural assumptions regarding the rationality and information in the supply chain can adversely affect the inherent uncertainty [6]. Games provide powerful frameworks to understand the interactions and decisions made by SC agents [8].

Various theoretical frameworks for supply chain studies have been developed. Broadly the studies can be classified into make or buy decisions, sourcing strategies, supplier selection decisions and negotiation and contracting interactions [3]. Games between supply chain agents can be studied as cooperative or non-cooperative games [9]. Non-cooperative games are strictly strategy-oriented detailing on what a player expects other players to do and precisely elaborating on the essentials of how to achieve the same. Our research draws inspiration from the scope and relevance of applying game theory tools to supply chains. While maintaining the focus on non-cooperative games, our study proposes a game model in the presence of uncertainty. This uncertainty is characterised by the impact of compromised rationality and incomplete information in the game structure. We measure uncertainty as the entropy of information [10], since information flow between agents is critical to the success of the SC network.

Theories have always played an important role for understanding organisational operations and have contributed to the economy. In section 1.1 we discuss the literature on rationality and information relevant to our study to understand the basic theoretical underpinnings of our proposed model following which section 2 elaborates on the development of the model. Application of the proposed model on a manufacturer – supplier game is illustrated in section 3, while the results and discussions are covered in section 4.

1.1. Information, Rationality, and Equilibrium

Conflicts are visible across all domains of decision-making heightened by asymmetric information and irrational behaviours. Consider the regional conflict between Israel and -Palestine. During the first half of 21st century, arguments were focused on removal of Israeli occupation from the West Bank and stopping Palestinian terrorism [11]. When studied as a game, this conflict can result in two different games, differentiated by the payoffs for the same available strategies but varying behavior of Palestinians. The presence of two possible games arises only when Israel does not have information regarding the nature of Palestine. One game as a result of a radical behavior and the other attributed to a moderate behavior while assuming the Israelis continue to play with a moderate behavior in both the games. This change in nature of Palestinians changes the game from a Non-Prisoner's Dilemma under radical behavior to a Prisoner's Dilemma under a moderate behavior [11]. Such changes in the nature affecting the game structure are not just restricted to political conflicts but also business wars, economic conflicts and varied scenarios including social conflict scenarios such as the Battle of Sexes game. The fundamental Battle of Sexes game is well explained in Luce & Raiffa (1957) [12 p. 90-91]. When the same logic of incomplete information is applied to a Battle of Sexes game, two alternate games with different payoffs as a result of the nature of the players (whether interested or uninterested in the opponent) can also be formed. Similar type of alternate games can arise when any agent has incomplete information regarding other agents in the supply chain network of operations.

Incomplete information regarding the opponent, gives rise to alternate games where the payoffs for the same strategy differ based on the nature of the players [11]. A player must act upon a strategy only when he is satisfied with the available information of his opponent [13]. The above statements might falsely lead one to believe that more information is beneficial in a game scenario. Kitti and Mallozi [14] throw light on the impact of more information on various equilibrium solutions. On the other hand, Atkinson et. al. (2016) [15] added the concept of waiting time while studying information in games. When relying on information before engaging with the target, the waiting time becomes critical. Players tend to incur costs if any action is taken based on wrong/incomplete information or by waiting too long in the hopes of gaining knowledge of the opponent's game. Engaging a target based on misinformation can result in collateral damage, loss of informers and may even result in misjudging the actions of the opponents.

In the case of complete information games, a measure of the cost of uncertainty associated with a player trying to trick the opponent can be considered in the game structure [8]. However, uncertainty in case of incomplete information is inherent in the structure since players are unaware of their opponent's nature. Thus, we can say that uncertainty exists in identifying the nature of the player and then in the basic structure of the game when the players engage in the game.

There are perhaps few issues in the theory of games that have generated as much discourse as that of rationality. The assumption, that players play rationally, is what engenders this most profound irony [16]. As early as 1955, Herbert A. Simon [17] had formulated a formal concept of rationality which entailed that a rational man should have knowledge, not necessarily complete but clear and voluminous enough, of the germane features of his environment. Also, that a rational man should possess a carefully designed stable system of preferences and the capacity to calculate which of the accessible courses of action will give him the highest attainability on his preference scale [17]. This approach organically led to applications of rational choice analysis to social issues quite beside the production and distribution of material goods [18].

Amongst various solution concepts developed by theorists over the years, Nash's theory of Non Cooperative games [19, 20] became a major breakthrough in studying rational choice analysis in general competitive scenarios and the importance of rationality as a parallel to attain equilibrium solution came into light. While the Nash solution was popular for developing traditional solutions, theorists have argued how Nash equilibrium is neither a necessary nor a sufficient property [21, 22] as a criterion to identify the optimal strategy for the players in a game. This draws from the understanding that rationality in itself is a disputed concept in the theory of decision making.

Simply put, the concept of rationality assumes that every subject is spurred on by the maximisation of one's own reward. The consistency of an individual's decision-making in the presence of different sets of available actions is the proper way to measure an individual's "rationality" without considering his likes and dislikes [23]. The theory of games rely heavily on the assumption of rationality and the theory of rational choice can be found as a unit in many a model of game theory. Many solution concepts that have been developed possess both qualities of rationality and equilibrium. Such solutions are applicable only in an idealistic setup of a game which gets easily disturbed as soon as the structure of the game accommodates variability in the form of number of players or payoffs. Rationality and equilibrium then cannot be achieved simultaneously [24].

A number of authors have weighed and debated the importance of rationality over equilibrium and vice-versa. It is not necessary that agents will always consider the opponents objectives in fact they may damage them if necessary. The possibility of such a scenario is highly influenced from the tit for tat behavior, i.e. people end up treating others how they are being treated. In the process some players don't hesitate to hurt their opponents while forgoing their own wellbeing if necessary [25]. There have been other authors as well who have doubted the applicability of rationality stating that it may be difficult to accomplish in reality [26, 27]. The most pressing issue arises from the fact that lab experiments indicate that players time and again fail to conform to the basic assumption of rationality in game theory [28, 29]. In addition, rational analysis (and not just in the form of assumption) has often failed to be consistent with reality. Even through simple introspection, complete rationality seems faulty and implausible [16, 30]. Many authors have taken a departure from the conventional theory of rationality and re - defined rationality to propose approaches that explain the real behaviours of players [31, 32, 33].

One such cautious way to study rationality is the concept of bounded rationality acknowledging that rationality also fails. Bounded rationality is an easier approach to incorporating rationality as it allows players to decide based on their satisfaction level instead of forcing one to find the optimal value. If players operate from a level of satisfaction, then there exists no need to display rationality to arrive at a Nash equilibrium. Conlisk (1996) [34] furthers the discussion on bounded rationality highlighting the popularity of bounded rationality in the presence of already established models of complete rationality. His work highlights that (i) there is enough evidence in the field of Psychology and economics on the importance of bounded rationality, (ii) the models developed on the premise of bounded rationality have been successful in demonstrating economic behaviour beyond just theory, (iii) traditional methodologies in economics favours bounded as well as unbounded rationality and (iv) respecting the fundamental concept of limited resource and hence expending the same in an efficient manner. Human cognition in this case can be considered to be such a resource. Given these principles, bounded rationality has found applications over the years across various disciplines [35, 36, 37, 38].

Another approach to rationality is rationalizability. Both Bernheim [22] and Pearce [39] introduced rationalizability in their work highlighting the need of common knowledge for rationalizable outcomes. Rationalizability is noted as the consequence to two premises based on which strategic situations can be interpreted. First, players view opponents' strategies as uncertain events; Second, all agents abide by individual rationality and this fact is common knowledge. The second premise requires a player's assessment to be consistent with all the knowledge that the player has about the game. Thus, the player must not only be aware of the opponent's choices but also be able to attach a positive value to the predicted play of the opponent. Further, the player should also have some belief regarding the opponent's assessment of the player's action which would result in a best response from the opponent as well. Since this is common knowledge for all the players, this can be extended indefinitely. This implied that to display rational behaviour rationalizability was an important criterion but not Nash equilibrium. Further, if players are cautious and not only rational, it would result in stringer rationalizability. Thus, attempts to exploit the information structure – be it introducing bounds on the payoffs or predict a specific outcome must be attempted when analyzing a game.

Given the stringent assumptions of the Nash solution concept, one has to search for more flexible solution approaches that account for imperfect behaviour [40]. Imperfect behaviour in simultaneous games can be explained through risk seeking behaviour of the players. It accounts for the costs associated with such a risk in the absence of any lucrative advantage [39]. Similar to such imperfections, Sengupta and Panandikar (2023) [10] have modelled cost of uncertainty into the payoff structure of simultaneous games, to show that players are willing to take the risk and deviate from strict rational behaviour if it increases the uncertainty for the opponent.

Our study, focuses on the concept of bounded rationality and rationalizability in contrast to strict rationality and draws relevance from these concepts to explain the results of the model. In the absence of complete information, rationalizability will indicate that a player must be equally cautious as he is rational, since he is unaware of his opponent's behaviour, which reduces the predictability. If we assume the player to operate from a space of bounded rationality, player is aware of the limited resources available to self and thus cannot afford a search for his optimal strategy indefinitely. Hence, such decision-makers have to find a tradeoff between the utility he will obtain from his strategy and the resource cost of actually locating the optimal strategy [41]. This is the premise for the proposed model of our study.

2. Proposed Game under Incomplete Information

Consider a dating game (as a corollary to the Battle of Sexes (BoS) game as introduced in section 1) under the complete information scenario. The main aim of the couple (players) is to be together at a chosen event, irrespective of their individual event preferences. However, the objective may change the moment any one of the given players have incomplete information. In this scenario incomplete information implies that a given player is unaware of the willingness of the opponent to be together. This will give rise to different set of payoffs – one if the partner is willing to be together more than individual's event preference and another if the partner is unwilling to be together given the individual event preferences.

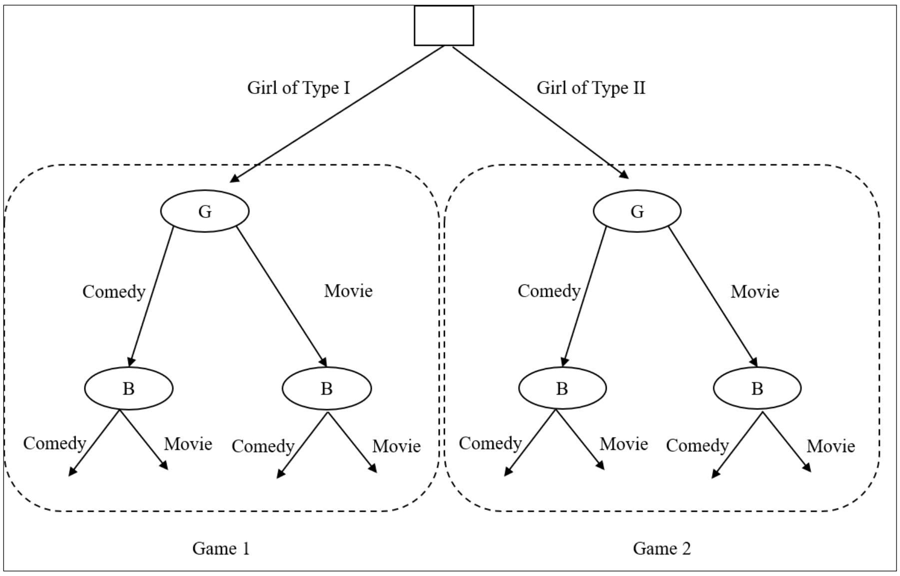

Figure 1 BoS game structure in the presence of incomplete information

Figure 1 depicts this incomplete information scenario for a dating game between player I and player II. Player I has incomplete information regarding player II's type and interest in player I. Player II has complete information regarding player I's interest in player II. Assume that the two events to be chosen from are a comedy show and a movie in the theatre. Player I's preference is a comedy show while that of Player II is movie in the theatre. Incomplete information regarding the type of the player II escalates the dilemma for player I as it gives rise to alternate games, each with the strategy space of {Comedy Show, Movie}. Game 1 in figure 1 refers to the scenario when player II is interested (type I) in player I similar to the base model of a BoS game. Game 2 refers to the scenario when player II is not interested (type II) in player I. In this illustration we have assumed only two alternate types of Player II resulting in two alternate games with different payoff structure.

BoS game was used to introduce the concept of alternate games under incomplete information. In the remainder of the study, we continue to refer to 2 person games. In our proposed model we assume player II has complete information about player I and player I has incomplete information regarding the type of player II. 'Type' of the player is defined as the nature of the player allowing for its applicability across all types of games. Each type corresponds to a game with a different payoff structure. Hence, the number of alternate games will be equal to the number of types of player II. In our proposed model, we clearly state the role of information and entropy in the structure of the game.

2.1. Game Structure

We propose a game structure for two person bi-matrix games under incomplete information. We account for the inherent uncertainty owing to the presence of alternate games in the form of entropy of information.

Players collect information regarding the opponent without communicating with them directly, with the hope of reducing the uncertainty present in the game. All information is collected prior to engaging in the game. No information will be collected once the players engage in the game. There will be cost associated with the accumulation of information. This cost of information will be dependent on the uncertainty present in the information and will be measured as entropy of information. The authors measure the uncertainty present in the information in terms of Shannon entropy [42]. Entropy is accounted for at two different levels, once before engaging in the game and the second time during the game. During the game we include the entropy of information as defined by Sengupta and Panandikar (2023) [10] in their model. Player I needs to account for the cost associated with entropy before engaging in the game and during the game, while player II is concerned with the impact of the cost of entropy incurred during the game only.

A step wise explanation of the structure of the game along with assumptions has been given to understand the mathematical development of our proposed model.

- The total number of alternate games will be equal to the total number of varied types of player II. The actual type of Player II is denoted by τ $(\tau = 1,2,\cdots k)$.
- There is a probability associated with each of the type of Player II, i.e. $(P(\tau = t) = p_t\ , t = 1,2 \cdots k)$. Eventually, player II will play any one of the k probable games basis his nature which player I is unaware of.
- Player I is unaware of the type of player II. However, player I has a belief system about the type of his opponent that is built on the basis of the information he collects. He then plays his game, based on this belief system. The belief system of the player I is denoted by β, and is given by a set of probabilities associated with the k games of the opponent.
- Impact of uncertainty on player I: Player I accounts for uncertainty at two levels, once before engaging in the game and secondly during the game.
    - *Before engaging in the game*: Player I is relying on his belief system regarding player II. Before engaging in the game, he has the freedom to collect information regarding the type of player II. The collected information helps to reaffirm his belief regarding his opponent. In the process, he either gets reassured or he changes his belief. Continuing the process, player I updates his belief system after every set of evidence collected, with the hope of being able to predict which of the k games player II would finally engage in. Uncertainty in this phase is characterized by player I's belief of player II's type given by the set of probabilities $(P(\beta = b) = p_b, \forall b = 1,2,\cdots k)$. Player I engages in the game only after he has decided to stop collecting information regarding the type of player I. The final decision to engage at any time point is dependent on the updated belief system. The updated belief system relies on the latest available information. However, resources used to collect evidences incur cost. Therefore, cost of information is accumulated over all the rounds of collecting information, till player I decides to terminate collecting information and engage in the game.
    - *During the game:* During the game, uncertainty is associated with the strategies available to player II for a given game. This uncertainty impacts the payoff of player I only after player I has stopped collecting information regarding opponent's type and decides to engage with player II in a given game $(\beta = b)$ as identified by his belief system. Here, the information is characterized by the probability of the j^{th} strategy of the b^{th} game of player II i.e $(P\big(Y = y_j\big)_{\beta=b} = y_{jb})$.

- Impact of uncertainty on player II: Player II has complete knowledge regarding Player I, i.e., Player I's type is known. and hence information is collected only regarding the strategy space of player I. Value of information is characterised by the i^{th} strategy of player I's game ($P(X = x_i) = x_i$).

2.2. Engage in the Game

Since cost of collecting information keeps accumulating, while player I keeps updating his belief system, a check on the process of collecting information becomes extremely important. Hence our model incorporates the process to identify when player I should stop collecting information and engage in the game.

- To avoid any loss, a player will collect information as long as his resources are sufficient and he is gaining monetary benefit from the whole transaction.
- If resources are abundant, a player would terminate collecting information if he reaches a saturation of his belief system with respect to the type of the opponent.

A rational player, would stop accumulating information and engage in the game as soon as the player realizes that his (her) resources are depleting to a level of monetary loss. Accumulation of excess information can earn losses for player I if the cost is not kept under control. However, this is the trade-off that a player, playing within his boundaries of rationality, is ready to accept in his payoff hoping to play the game with a more informed decision. So, the decision to stop accumulating information is now entirely dependent on the saturation of his belief system.

Assumptions regarding the belief system of Player I:

- Belief system of player I is updated every time evidence is collected. Let $l = (1,2,3,\cdots n)$ be the time point at which evidence (information) is collected. Then, player I's belief about player II's type is defined as a stochastic process, denoted by $\{\beta_l\}$.
- Updating the belief system depends only on the current set of collected evidence.
- Probability of updating the belief at any given stage is independent of when the evidence is collected.

Assumption 1 implies that collecting information is a discrete time process. Now, consider β_l as defined above, then the state space denoted by S is a finite state space defined by the type of the player i.e. $S = \{1,2,\cdots l\}$. The proposed game structure combined with assumption 2, emphasizes that the belief system at n^{th} stage is updated to reaffirm the belief at $n-1^{th}$ stage. Hence change in player I's belief regarding Player II's type, from β_{n-1} to β_n, depends only on his belief at $n-1^{th}$ stage, thus β_l also satisfies the markov property. Thus $\{\beta_l\}$ can be interpreted as a discrete markov chain (Isaacson & Madsen, 1976, pp. 12-14).

Assumption 2 and 3 combined assure that probability of updating belief from type i to type j is independent of when we update the belief, i.e.,

$$P[\beta_n = j \setminus \beta_{n-1} = i] = P[\beta_{n+l} = j \setminus \beta_{n+l-1} = i]$$

Thus, to map the saturation of the player I's belief regarding player II, a transition matrix P^1 is defined, such that ${p_{ij}}^1$ is the probability that player I's belief regarding the type of player II will change from type i to type j on collecting the first set of evidence. Hence, P^n gives the probability that player I's belief regarding the type of player II will change from type i to type j after collecting n^{th} set of evidence.

Definition 1 *(Saturation of Belief System): Mathematically, Saturation of Belief System is defined as the state when the conditional probability obtained at any future stage is equal to the conditional probabilities obtained at* n^{th} *stage for all states of the belief system.*

In other words, the transition matrix P^{n+m} should be equal to P^n $\forall$ $m = 1,2,\cdots$. Hence, we can say that the belief system is saturated if $P^n \cong P^{n+1}$. Thus, at $n+1$ stage player I terminates collecting information and decides to engage in the game.

Therefore, we can say that player II's type is characterized by the set of probabilities given by $\{\beta_l\}$ at any given stage after player I has sourced the required information. We can say that the uncertainty at a given time point l will be a function of the updated probabilities of $\{\beta_l\}$. We denote the probabilities at a given time point l as $P(\beta_l = b) = {p_b}^l, \forall\ b = 1,2\cdots k\ types$. The Shannon entropy used to capture the above uncertainty in our model is given by equation (1).

$$H_b{}^l = -\sum_{b=1}^{k} p_b^l \log_2 p_b^l \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

The three assumptions above identify the belief system as a Discrete Markov Process. If player I's belief system saturates after n evidences of information has been collected, i.e. $P^{n-1} \cong P^n$, then there will be a total of n entropy measures given by the equation (1) accounting for the uncertainty in the information collected at every stage. We model the cost of information as a function of the Shannon entropy [10]. Equation (2) gives the cost of information related to the uncertainty associated with the type of player II. This cost is generated after every round of evidence is collected. Total accumulated cost when player I saturates his belief system after n evidences is calculated as an aggregate of all the individual costs defined by equation (3).

$$C_b{}^l = \lambda + \mu * H_b{}^l \quad \forall\, l = 1,2, \cdots n \quad \ldots\ldots\ldots\ldots\ldots (2)$$

$$C_b = \sum_{l=1}^{n} C_b{}^l \quad \ldots\ldots\ldots\ldots\ldots (3)$$

Therefore, when player I terminates collecting information, his belief system is updated to the latest probability distribution given by $\{\beta_n\}$ and he has already accumulated a cost as given by equation (3).

When player I engages in the game, additional cost attached to the uncertainty about the strategies of player II's b^{th} game will also be a function of the Shannon entropy. During the game, player II must also account for the cost associated with the strategy space of player I. Assume that player I and player II have a total of m strategies to engage with. Then the entropy of the strategy space associated with player I and II is given by the equations (4) and (5) respectively.

$$H_x = -x_i \sum_{i=1}^{m} \log_2 x_i \ldots\ldots\ldots\ldots\ldots (4)$$

$$H_y = -y_{jb} \sum_{j=1}^{m} \log_2 y_{jb} \quad \ldots\ldots\ldots\ldots\ldots (5)$$

Player I would have already accumulated C_b as the total cost before engaging in the game. The additional cost during the game is captured using equation (6) for player I. Equation (7) denotes the total cost for player II.

$$C_x = C_b + \mu * H_x \ldots\ldots\ldots\ldots\ldots (6)$$

$$C_y = \lambda + \mu * H_y \quad \ldots\ldots\ldots\ldots\ldots (7)$$

We model the incomplete information game as a non-linear bi-matrix game that incorporates the cost of entropy in the model. System of equations given by (8) and (10) are the bi-matrix game models developed for player I and player II respectively. Obtaining the equilibrium solution to these non-linear problems simultaneously will give the optimal solution for both the players.

Model:

For player 1,

$$\text{Maximixe } x^T A y_b - C_x \ldots\ldots\ldots\ldots\ldots (8)$$

Subject to

$$e'x - 1 = 0 \quad \ldots\ldots\ldots\ldots\ldots (9a)$$
$$x \geq 0 \quad \ldots\ldots\ldots\ldots\ldots \quad (9b)$$

For player 2,

$$\text{Maximixe } x^T B y_b - C_y \quad \ldots\ldots\ldots\ldots\ldots (10)$$

Subject to

$$e'y - 1 = 0 \ldots\ldots\ldots\ldots\ldots (11a)$$
$$y \geq 0 \quad \ldots\ldots\ldots\ldots\ldots (11b)$$

Once the players decide to engage in the game, player II will play any one of the probable k games basis his actual type. According to player I, player II's probability of playing the b^{th} game is given by p_b^n. The expected payoff to player I, given that player II is playing his b^{th} type is given by (8). There exists k such possibilities and the resultant payoff will depend on the probability of the b^{th} game as well.

There is a possibility of any one of the k types being played. Therefore, for a given b^{th} type, the resultant payoff obtained to player I can be given as $p_b^n * (x^T A y_b) - C_x$. For every game, the equilibrium solution will be defined by a pair of strategies x^0 and y^0, if (x^0, y^0) simultaneously optimizes (8) and (10). Thus, the solution to equations (12) and (13) will result in a local optimal solution for the specific b^{th} game.

$$x^{T0} A y_b{}^0 - C_x = \max_x \{x^T A y_b - C_x \mid \quad e'x - 1 = 0,\ x \geq 0\} \quad \text{………….. (12)}$$

$$x^{T0} B y_b{}^0 - C_y = \max_y \{x^T B y_b - C_y \mid \quad e'y - 1 = 0,\ x \geq 0\} \quad \text{………….. (13)}$$

There will be a total of k optimal solutions. To capture player I's total earnings, expected payoff across all the k games need to be calculated.

The proposed game structure under incomplete information games has been illustrated using a single manufacturer – supplier relationship in the following section. The illustration helps us understand how much a player would be willing to trade off the profit against the cost of information.

3. Application of the Proposed Game under Incomplete Information

In section 2, we have identified player I to have incomplete information regarding player II. However, player I has a belief system regarding player II and this belief system has been modelled as a Markov Chain to obtain the probability distribution of the opponent. We first explain the process to identify the probabilities related to the belief system of player I and then apply the same on a single manufacturer – single supplier scenario.

P^n was defined as the probability that player I's belief regarding the type of player II will change from type i to type j after collecting n^{th} set of evidence. This probability is given in terms of a probability transition matrix. Assume a scenario given by the transition probability P^1 represented in table 1. It accounts for the belief that player I has after collecting the first set of evidence regarding player II. The values in the table should be read as the probability that player II will change from type i to type j as per player I's belief, basis the evidence collected.

Table 1 Transition Probability matrix $\boldsymbol{P^1}$

$P^1 =$		jth type	
		Type 1	Type 2
i th type	Type 1	0.65	0.35
	Type 2	0.85	0.15

After collecting the first set of evidence, the probability structure will be as given by P^1. After collecting the second set of evidence, updated belief would be given by $P^2 = P^1 \times P^1$. Thus, the belief system is updated as a markov process at every stage. The belief after the n^{th} set of evidence can be given as $P^n = P^1$ multiplied by itself n times.

This accumulation is stopped once the saturation of belief system is attained that is $P^n \cong P^{n+1}$. For the given values of P^1 in table 1, saturation is obtained at P^8 correct to 4 decimal places as shown in table 2. This implies that 8 sets of evidence regarding the type of the player was obtained to reach saturation. And at every step the belief system was updated. The final probability structure in this case is given by P^8. The initial belief structure given by the transition probability matrix P^1 weighed heavily on type 1 hence, the saturation structure also resulted in higher probability towards type I. Different starting scenarios will result in different saturated belief system. We have chosen P^1as given by table 1 as the scenario to elaborate on for our study.

Table 2 Step wise update of player I's belief system

			j	
Transition step P^n			Type 1	Type 2
P^2	i	Type 1	0.72	0.28
		Type 2	0.68	0.32
P^3	i	Type 1	0.706	0.294
		Type 2	0.714	0.286
P^4	i	Type 1	0.7088	0.2912
		Type 2	0.7072	0.2928
P^5	i	Type 1	0.70824	0.29176
		Type 2	0.70856	0.29144
P^6	i	Type 1	0.70835	0.29165
		Type 2	0.70829	0.29171
P^7	i	Type 1	0.70833	0.29167
		Type 2	0.70834	0.29166
P^8	i	Type 1	0.70833	0.29167
		Type 2	0.70833	0.29167
P^9	i	Type 1	0.70833	0.29167
		Type 2	0.70833	0.29167

At this stage player I updates his belief system using the saturated matrix P^8 and then engages in that game. The equilibrium solutions are obtained for both the games, after incorporating the uncertainty in the system in the form of costs as discussed in section 2.2 and the resultant payoff of player I is obtained by considering the expected probability over the types using the distribution given by β_8. Following which, the resultant payoff in both scenarios is conducted.

3.1. Single Manufacturer, single supplier

Uncertainties in demand leads to faulty demand forecasts which leads to inefficient handling of demand overall [12]. Inaccuracies and disruptions in the supply chain are a major concern across industries such as automotives, machineries and even agriculture and healthcare [43, 44, 45, 46, 47]. Hence, we extend our study to understand the application of the same in an example of transactions between a single manufacturer and a single supplier with respect to demand forecast.

Consider a model with one supplier and one manufacturer. A manufacturer sells a single product that has uncertain demand. The manufacturer is assumed to contract with a single supplier for a specialized component of the product. The supplier has to build capacity for this specialized component. This example draws from the design proposed by [48]. The supplier is expected to install capacity before either of the parties observe demand. The manufacturer is observed to have better demand forecast than the supplier. Thus, the supplier must rely on the communication received by the manufacturer regarding the forecasted demand. However, supplier is unaware of the type of the manufacturer giving rise to alternate games. The following section explains the sequence of events in the single manufacturer – single supplier game model.

First, the manufacturer observes the demand distribution, which is recorded as forecasted demand (Q). Then manufacturer offers contract to supplier with an initial order quantity (q_i) basis Q. Once the supplier accepts the contract, he sets a capacity K in accordance to the manufacturer's order q_i. Then the final number of units (q_f) are ordered once the final production takes place. In an ideal world a manufacturer would truthfully share her demand forecast so that the supplier can build an appropriate space. However, the larger the capacity, manufacturer will benefit from it in case of high actual demands but the supplier has to bear the cost of capacity [2, 48]. Hence, the manufacturer finds motivation in inflating her forecast to the supplier. However, suppliers are aware of the possible distortion in the communication and thus view the forecast with some skepticism (which may be reduced if the forecasts are backed by contract terms assuring credibility of the manufacturer).

The type of manufacturer is characterised by the communicated demand forecast. While a supplier is aware of the possible forecasts, he is still not aware of the real type of the manufacturer. Thus, the supplier has 2 strategies available that is to build a high-capacity installation (H) or a low capacity installation (L). For a high capacity, the cost of installation will be extra if the final number of units ordered is considerably lesser than predicted. This situation worsens if the forecast is an inflated one. Hence, we assume that the capacity size for H should not be more than the inflated forecast to help the supplier meet his basic opportunity cost. Likewise, an extremely low capacity will always result in some units being undelivered and affect the profit as well as good will. Hence, we assume that the capacity size for L should not be less than the final order of the actual forecast.

Table 3 mentions the assumed values and notations used for this application to arrive at the equilibrium solution of the game.

Description	**Notation / Values**
Forecasted Demand	Q
Initial Order	q_i
Final Order	q_f
Price per unit of raw component that is supplied to the manufacturer	0.75
Cost of installation capacity per unit	0.1
Cost of production per unit to supplier for preparing the components that are to be delivered once the final order is received	0.1
Revenue to manufacturer per unit	1
Supplier’s installation Capacity in terms of number of units to be stored	K units
Total number of components produced that can be delivered to the manufacturer.	P units

Here, P will be either less than or equal to q_f. Since there is a single supplier providing the units to the manufacturer, if P number of components are delivered to the manufacturer, the total number of finished products delivered by the manufacturer will be P units itself. The payoff functions for the supplier and the manufacturer are then defined by equations (14) and (15) respectively:

$$\text{Payoff to the supplier} = 0.75 * P - 0.1 * K - 0.1 * P \quad \ldots\ldots\ldots\ldots (14)$$

$$\text{Payoff to the manufacturer} = 1 * P - 0.75 * P \quad \ldots\ldots\ldots\ldots (15)$$

Table 4 and 5 are the payoff matrices for the alternate games arising from the two types of the manufacturer. If the manufacturer conveys the actual forecast to the supplier then we can say $Q = q_i$ and we denote this type with “ A ” referring to actual forecasted demand; but if the manufacturer inflates the forecast to the supplier then we assume that $Q < q_i$ and this type is denoted with “ I ” referring to the inflated communication of the forecasted demand. Assuming $Q = 10$, type “ A ” will imply $q_i = 10$ and for type “ I ” we assume a larger value, i.e. $q_i = 15$.

Table 4 Payoff structure for Game 1 – Type “ A ” ($\boldsymbol{Q = 10, q_i = 10}$)

		Manufacturer	
		$q_f = q_i$ (10)	$q_f < q_i$ (5)
Supplier	H (15)	(5, 2.5)	(1.75, 1.25)
	L (5)	(2.75, 1.25)	(2.75, 1.25)

Table 5 Payoff structure for Game 2 – Type " I " ($\boldsymbol{Q = 10, q_i = 15}$)

		Manufacturer	
		$q_f = q_i$ (15)	$q_f < q_i$ (10)
Supplier	H (15)	(8.25, 3.75)	(5, 2.5)
	L (5)	(2.75, 1.25)	(2.75, 1.25)

Whereas $q_i = 10$ and $q_i = 15$ represent the initial order quantity for both the type of games, the transaction takes place on the final order quantity (q_f). q_f maybe same as the initial order quantity or lesser as per actual market demand. Higher market demand is not captured separately since inflated forecast takes care of the same. Thus, the manufacturer's strategies will be defined in terms of the final order quantity as either $q_f = q_i$ or $q_f < q_i$. The value of q_f will vary for both the games as q_i varies. The assumed values for q_f have been mentioned in brackets for both the games as given in the payoff matrices depicted by table 4 and 5.

The strategies available to the supplier is to either build a high or low installation capacity and that is based on the initial communication from the manufacturer. Adhering to the capacity size assumptions mentioned earlier in this section, extreme values for H and L have been considered to depict that H cannot be more than 15 and L cannot be lesser than 5. We use the above game parameters in the payoff equations (14) and (15). This generates the payoff values given in the payoff matrix tables 4 and 5.

The supplier is going to collect information regarding the manufacturer before installing the capacity. The theoretical underpinning of the same is as discussed in chapter 4 and is a result of the belief system the supplier has. Belief saturation using transition probability matrix as explained under section 4 is implemented in this example. Recall that the supplier is the one with incomplete information and he starts with a set of belief or probability structure regarding manufacturer's types and engages in the game once this belief saturates. We use the probability matrix P^1 and hence the saturated probability matrix P^8 for further analysis of the game.

We have obtained, $P^8 = \begin{bmatrix} 0.7083 & 0.2917 \\ 0.7083 & 0.2917 \end{bmatrix}$ (table 2) and can be interpreted as there is 70.83% chance the manufacturer is type A while only a 29.17% chance manufacturer is type I. Thus, $\beta_8 = [0.7083 \quad 0.2917]$. The cost of accumulation of evidence is calculated to be 8.52. Table 6 records the output for both the types. For the cost of entropy, we assume the fixed cost associated with entropy is marked at 1 unit and that associated with the variable cost is marked at 0.75 units per entropy.

Table 6 Comparative output of TYPE A versus TYPE I

i or ***j***	$\boldsymbol{x_i}$	$\boldsymbol{y_j}$
TYPE A		
1	0.5641012	0.5365912
2	0.4358988	0.4634088
Entropy (H)	0.9881113	0.9961332
Expected payoff	6.227891	4.551307
Cost associated with entropy	9.261862475	1.7470999
TYPE I		
1	0.8085493	0.5866848
2	0.1914507	0.4133152
Entropy (H)	0.7044964	0.9782085
Expected payoff	8.601932	5.892079
Cost associated with entropy	9.04914513	1.733656375

4. Results and Discussion

The supplier is mainly concerned with knowing whether the forecasted demand is actual or the manufacturer has inflated it. Information plays an important role at this step. Once the supplier is convinced with the information he has acquired, he installs the capacity which is either equal to the initial order quantity or lesser which he decides as per contractual agreements (like forced compliance or voluntary compliance) and supplier capacity. However, the final order demand is influenced by the actual market requirement and falls into place after the supplier has installed the capacity. Hence, in this example we assume that collection of information is possible and plays a significant role only at the first step before setting up the facility.

As per the assumed data, we can see that the supplier has to incur loss in both scenarios owing to the high accumulated cost over information. If the manufacturer shares the actual forecast, the supplier receives a loss of 3.03 units but if the manufacturer inflates, the supplier receives a loss of 0.45 monetary units only. Collecting information increases the cost for him, but when the players engage they do so in a way such that the opponent remains confused. That is a typical scenario of tit for tat where players would treat others the way they believe they are getting treated as mentioned in the literature. Hence, the supplier has been able to reduce the loss in the case when he believes the manufacturer is inflating the demand.

If we focus on the belief system, we can say that based on the belief system, supplier believes that manufacturer will give him the actual forecast (approximately 71% probability). Given this belief we observe that the mixed strategy under Game 1 that supplier adopts shows how the supplier plans to increase the uncertainty for his opponent (56% probability of installing a High capacity production or a 44 % chance of a Low capacity production). The total cost associated with the Actual game is higher even though the expected payoff is lesser for the supplier with respect to this game. This only emphasizes the tit for tat behaviour where the supplier increases the uncertainty for his opponent (0.99), in the game with Actual Forecast type, to a level higher than the other game (0.97). Thus, there is a visible effort in trying to keep your opponent guessing and that is also driven by the fact that the supplier himself had incomplete information about the opponent. Thus the players are playing within their boundaries of rationality, and are ready to forgo benefits in the hope that the payoff of their opponent gets compromised.

When players engage in presence of distortion, ignoring its impact on the game worsens the performance of a player as it reduces the predictability. Hence, if the players were to acknowledge the uncertainty in the system caused by the probability of alternate games, it would help reduce collateral damages. The application discussed above was used to understand how the developed model can be used in an industrial application involving various agents of a supply chain. However, this application can be extended to other players and other domains to study the impact of alternate games. Our focus was on depicting the implementation of information in the structure of games and how it may impact the equilibrium and rational choices that a player would take otherwise in the absence of such measures. Using the entropy of information model to optimize the value of a game combined with the Markovian process of updating one's belief in the presence of incomplete information helped to capture the notion of human welfare that players are responsible for their individual actions within their domain of control.

5. Conclusion

A key finding of this study is that players strategically sacrifice a part of their payoff to minimize the uncertainty and misinformation about their opponent. This is contrary to the traditional assumptions of rationality but it supports the developments in the field of rationality in the later years as discussed in section 1.2. This study discussed a two person zero sum game in the presence of incomplete information and the application used helps to understand the possibility of alternate games arising from different types of the opponent. Business across various industries and the broader market often operate in environments where they are faced with making strategic moves with limited knowledge of their competitors. Hence, the proposed model helps to visualize the behaviour and actions of players in such a scenario and provides a direction of how much risk in the form of cost of information should a player be ready to take. This knowledge can aid businesses in refining strategies and gain competitive edge in uncertain market conditions. While in our proposed model, we have assumed that only one player has incomplete information, complexities will only increase if number of alternate games increase or both players have incomplete information about each other. While both the scenarios are outside the scope of this study, these along with sequential games form potential avenues for future research.

Compliance with ethical standards

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Disclosure of conflict of interest

The author(s) report that there are no competing interests to be declared.

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